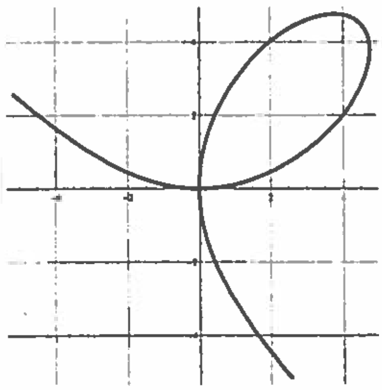
Department of Mathematics and Systems Analysis
Aalto University, School of Science
Metsalo

MS-A0210 Mathematics 1

Final exam, September 8, 2016

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Degree Programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

Calculators are not allowed. The examination time is 3 hours.
Ask, if You suspect typos in the text!



1. The curve $x^3 + y^3 = 9xy$ in the figure to the right is called *the Folium of Descartes*. It passes through the points $(0,0)$, $(4,2)$ and $(2,4)$ and has a horizontal (and a vertical) tangent at the origin and a horizontal tangent at another point (a,b) as well (near the point $(4,5)$, according to the graph).
   Determine the coordinates a and b of this other point, where the Folium of Descartes has a horizontal tangent.
2. Use the method of Lagrange multipliers to find the maximum value of $f(x,y,z) = xy^2z^3$ on the sphere $x^2 + y^2 + z^2 = 12$.
3. Calculate the area of the part of the surface $z = x^2 + y + 1$, that has the (right-angled) triangle with vertices $(0,0), (2,-4)$ and $(2,1)$ as its projection (down) onto the xy-plane.
4. The circular disk $D = \{(x,y) \in \mathbf{R}^2 | x^2 + y^2 \le R^2\}$ has at a point $(x,y) \in D$ area-density $\delta(x,y) = \delta_0 \cdot (1 - \frac{x^2}{R^2})$, so $\delta_{min} = \delta(\pm R, 0) = 0[kg/m^2]$ and $\delta_{max} = \delta(0,y) = \delta_0$.
   Calculate its mass m. (Check: $\delta_{min} \le \frac{m}{A} \le \delta_{max}$.)
5. Let us start with a circle C with radius $\frac{a}{2}$ and center $(-\frac{a}{2}, 0)$, as in the figure below to the left. We construct a family of circles, each with center on the circle C and passing through the origin, as in the figure below in the center. Show that this family of circles has the cardioid $r = a(1-\cos\theta)$ (*i.e.* $(x(\theta), y(\theta)) = (a(1-\cos\theta)\cos\theta, a(1-\cos\theta)\sin\theta)$) as its envelope.
   (Comment: The cardioid is also traced by a point on the boundary of a circle, which rolls on another circle of equal size, as in the figure below to the right, but that does not have to be shown here.)

Useful (?) formulas:
$(a+b)^2 = a^2 + 2ab + b^2, (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
$\cos^2 t + \sin^2 t = 1$, $\cos^2 t = (1 + \cos(2t))/2$, $\sin^2 t = (1 - \cos(2t))/2$,
$\sin(2t) = 2\sin t \cos t$, $\cos(2t) = \cos^2 t - \sin^2 t = 2\cos^2 t - 1 = 1 - 2\sin^2 t$.