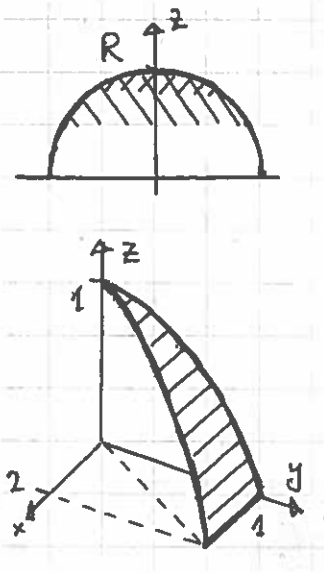
Department of Mathematics and Systems Analysis
Aalto University, School of Science Metsalo

MS-A0210 Mathematics 1

Final exam, 10.05.2017

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Degree Programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KTA, KON, MAR, MTE, PUU, RRT, TFM, TIK, TLT, TUO, YYT.

Calculators are not allowed. The examination time is 3 hours.
Ask, if You suspect typos in the text!



1. Show that the limit $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ does not exist.
2. For rectangular coordinates x, y and polar coordinates r, θ we have $x = r\cdot\cos\theta, y = r\cdot\sin\theta \Rightarrow \tan\theta = y/x$ (when $x \neq 0$) and $r^2 = x^2+y^2$. Show that $\frac{\partial r}{\partial y}\cdot\frac{\partial y}{\partial r} = \frac{\partial \theta}{\partial x}\cdot\frac{\partial x}{\partial \theta}$.
3. The halfball $W = \{(x,y,z) \in \mathbf{R}^3 | x^2+y^2+z^2 \leq R^2, z \geq 0\}$ has volume $V = \frac{2\pi r^3}{3}$. Its density at the point $(x,y,z) \in W$ is given by $\delta(x,y,z) = \delta_0 \cdot \frac{z^2}{R^2}$, so $\delta_{min} = \delta(x,y,0) = 0(kg/m^3)$ and $\delta_{max} = \delta(0,0,R) = \delta_0$. Calculate the mass m of W.
(Hint: The average density $\bar{\delta} = m/V$ satisfies $\delta_{min} \leq \bar{\delta} \leq \delta_{max}$.)
4. Calculate the area of the part of the parabolic cylinder $z = 1-y^2$, which is bounded by the xy-plane, the yz-plane and the plane $x = 2y$. (Hint: The area of the surface can not be less than the area of its projection onto a (coordinate) plane.)
5. Use the method of Lagrange multipliers to determine the shortest distance from the parabola $y = \frac{x}{2}\cdot(x+2)$ to the point $(x_0, y_0) = (-2, 2)$.

Useful (?) formulas:
$(a+b)^2 = a^2+2ab+b^2, (a+b)^3 = a^3+3a^2b+3ab^2+b^3$.
$\cos^2 t + \sin^2 t = 1$, $\cos^2 t = (1+\cos(2t))/2$, $\sin^2 t = (1-\cos(2t))/2$,
$\sin(2t) = 2\sin t\cos t$, $\cos(2t) = \cos^2 t - \sin^2 t = 2\cos^2 t - 1 = 1 - 2\sin^2 t$.