MS-C1420 Fourier analysis (Aalto University)
Turunen / Saari

First mid-term exam (30.9.2013)

Please fill in the required information onto each answer sheet.

Calculators and mathematical tables are not allowed.

This time you may use the Fourier inverse transform formula.

About grading: Every exam problem will be graded from 0 to 6 points. Harmless small errors do not prevent from getting maximal points. You will get points if your answer contains at least some information (relevant definitions, pictures, calculations etc) — empty answer is surely worth zero.

1. Find Fourier transform $\widehat{s} : \mathbb{R} \to \mathbb{C}$, of signal $s : \mathbb{R} \to \mathbb{C}$, where

$$s(t) = \begin{cases} 2, & \text{if } |t| < 3, \\ 0 & \text{otherwise.} \end{cases}$$

   Due to the time symmetry, $\widehat{s}$ in this problem is real-valued, so present your solution accordingly!

2. Prove that the Fourier transform preserves energy, meaning that

$$\|\widehat{s}\|^2 = \|s\|^2$$

   for signals $s : \mathbb{R} \to \mathbb{C}$.

3. Show that the convolution $r * s$ of signals $r, s : \mathbb{R} \to \mathbb{C}$ satisfies

$$\begin{aligned} (r * s)'(t) &= r' * s(t) \quad \text{and} \\ \widehat{r * s}\,(\nu) &= \widehat{r}(\nu)\,\widehat{s}(\nu). \end{aligned}$$

4. Let us consider signals $s_0, s_1, s_2, s_3, s_4 : \mathbb{R} \to \mathbb{C}$. where

$$\widehat{s_0} = s_1, \ \widehat{s_1} = s_2, \ \widehat{s_2} = s_3 \text{ ja } \widehat{s_3} = s_4.$$

   How are s_0 and s_4 related to each other, and why so?