PHYS-C0256 - Thermodynamics and Statistical Physics
Exam on February 21, 2024

6 problems - 30 points

1. (Concepts, 3 points) Write a brief explanation for each of the following:
(a) Grand-canonical ensemble
(b) Equipartition theorem
(c) Gibbs entropy

2. (Probabilities, 3 points) Five coins ($N = 5$) are tossed. Find the probability of having $n = 0, 1, 2, 3, 4$ or 5 faces. Draw the distribution $p(n)$.

3. (Ideal gas, 6 points) **(a)** One mole of an ideal gas is carried from temperature T_1 and molar volume V_1 to T_2, V_2. Show that the change in entropy is

$$\Delta S = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}, \tag{1}$$

where C_v is the heat capacity at constant volume. (3 points)

(b) An ideal gas is expanded adiabatically from (p_1, V_1) to (p_2, V_2). Then it is compressed isobarically to (p_2, V_1). Finally the pressure is increased to p_1 at constant volume V_1 (see Fig. 1). Show that the efficiency of the cycle is

$$\eta = 1 - \gamma \frac{(V_2/V_1) - 1}{(p_1/p_2) - 1}, \tag{2}$$

where $\gamma = C_p/C_v$. (3 points)

4. (Canonical distribution, partition function, 6 points) A certain magnetic system contains N independent molecules, each of which has four energy levels given by 0, $\Delta - g\mu B$, Δ, $\Delta + g\mu B$ (g is a constant). Write down the partition function and from that compute the Helmholtz function, internal energy and the heat capacity.

5. (Thermal conduction, 6 points) Derive the Wiedemann-Franz law for the relation between thermal conductance $G_{\rm th}$ and electrical conductance $G_{\rm T}$, $G_{\rm th} = \mathcal{L}_0 T G_{\rm T}$, for a normal metal – insulator -normal metal (NIN) tunnel junction. Here $\mathcal{L}_0$ is the Lorenz number. Hint: heat current in a tunnel junction is given by

$$\dot{Q}_L = \frac{1}{e^2 R_T} \int d\epsilon (\epsilon - eV)[f_L(\epsilon - eV) - f_R(\epsilon)] \quad \text{and} \quad \int_{-\infty}^{\infty} dx\, x^2 \frac{1}{1+e^x} \frac{1}{1+e^{-x}} = \frac{\pi^2}{3}.$$

6. (Qubit relaxation, 6 points) Consider a two-level system (TLS) with energies $E_g = -E/2$ and $E_e = +E/2$ and populations p_g and p_e of the ground (g) and excited (e) states, respectively.

(a) Assume that the TLS is initiated in the excited state (non-equilibrium state) at time $t = 0$, that is $p_g(0) = 0$. Let the transition rate from the ground state to the excited state be $\Gamma_\uparrow$ and that from the excited state to the ground state $\Gamma_\downarrow$. Find the time evolution of $p_g(t)$. (2 points)

(b) What is the expectation value of power $P(t)$ emitted by the TLS to the environment as a function of time t. (2 points)

(c) Assume the TLS represents a qubit. Let us drive it initially to a superposition state $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)$. Find the time evolution of the density matrix components $\rho_{gg}(t)$ and $\rho_{ge}(t)$, with the initial condition $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$ assuming they obey the standard master equation due to coupling to the bath at temperature T. (2 points)

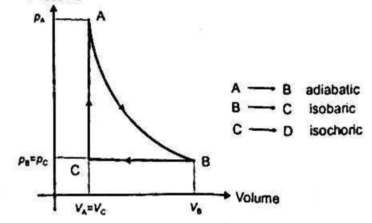

FIG. 1. Cycle of problem 3.