1. The wave function $\psi(\boldsymbol{r})$ of the electron in a hydrogen atom should satisfy the differential eigenvalue problem

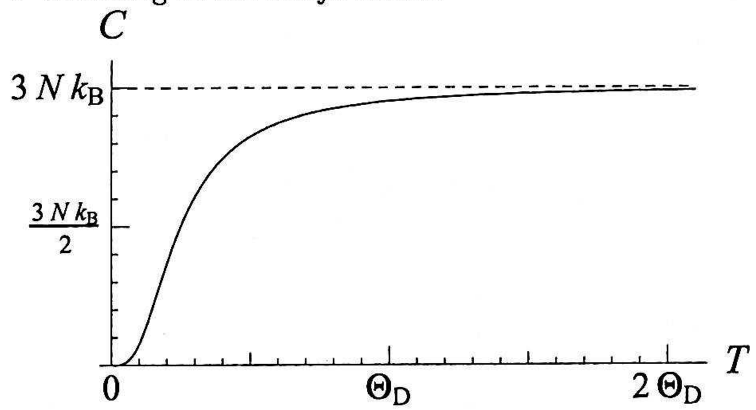
$$-\frac{\hbar^2}{2m}\nabla^2\psi(\boldsymbol{r}) + \frac{e^2}{4\pi\epsilon_0}\frac{1}{r}\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}). \tag{1}$$

Making the ansatz $\psi(r,\theta,\phi) = e^{\alpha r}$, determine the constant α and the energy eigenvalue E.

2.
   a) Discuss the Pauli exclusion principle: (i) what does it say about electrons filling the atomic orbitals? (ii) How is it formulated as a symmetry of the many-particle wave function?

   b) What makes the solution of the two-electron problem (as in helium atom) more complicated than the solution of electron levels in a hydrogen atom? What kind of approximations can be made in studying electronic structure of atoms with many electrons?

3. The figure below shows the heat capacity C of a solid material as a function of temperature T according to the Debye model.



   a) How can we understand the value of the heat capacity at high temperatures?

   b) How can we understand the diminished value of the heat capacity at low temperatures?

   c) The Debye temperature of silicon is 625 K. Estimate the specific heat of silicon per mole at room temperature.

4. We study magnesium using the free electron model. The conduction electron density is 8.61×10^{22} cm^{-3}. In the lectures we derived that the number of levels (per volume), whose energy is less that E:

$$n(E) = \frac{2\sqrt{2m^3}}{3\pi^2\hbar^3}E^{3/2}. \tag{2}$$

We assume the electron distribution corresponding to zero temperature. (a) Describe the distribution of electrons as a function of energy. (b) Describe the distribution of electrons as a function of $\boldsymbol{k}$ (wave vector in three dimensional space). (c) What is largest speed of the electrons? Give also numerical values of energy in (a) and velocity in (c).

5. Describe the energy bands of a semiconductor. How does temperature affect the electrical conductivity of semiconductors? How can we change the conductance properties by doping (inserting specific types of impurity atoms)? What is "recombination"?

Collection of formulas

Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t}(\boldsymbol{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\boldsymbol{r},t) + U(\boldsymbol{r})\Psi(\boldsymbol{r},t), \tag{3}$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\boldsymbol{r}) + U(\boldsymbol{r})\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}). \tag{4}$$

Spherical coordinates (r,θ,ϕ):

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}. \tag{5}$$

h, Planck's constant, $\hbar = h/2\pi = 1.054 \times 10^{-34}$ Js
$c = 299\,792\,458\,\mathrm{m/s}$, velocity of light in vacuum
$e = 1.602 \times 10^{-19}$ C, the elementary charge
$k_\mathrm{B} = 1.380 \times 10^{-23}$ J/K, Boltzmann constant
$N_\mathrm{A} = 6.022 \times 10^{23}$ 1/mol, Avogadro constant
$\mathrm{u} = (0.001\ \mathrm{kg/mol})/N_\mathrm{A} = 1.660 \times 10^{-27}$ kg, atomic mass unit
$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/Nm^2}$, electric constant
$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2} = 4\pi \times 10^{-7}\ \mathrm{T^2m^3/J}$, magnetic constant
$m_e = 9.109 \times 10^{-31}$ kg, mass of an electron
$m_p = 1.6726 \times 10^{-27}$ kg, mass of a proton
1 eV$= 1.602 \times 10^{-19}$ J