1. The Schrödinger equation for a harmonic oscillator is

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + \frac{1}{2}Kx^2\psi(x) = E\psi(x). \tag{1}$$

Look for a solution of this equation in the form $\psi(x) = e^{-\alpha x^2}$. Determine the constant α and the energy eigenvalue.

2. In the course we found that the orbital angular momentum in a spherically symmetric system (like the hydrogen atom) can be described by two quantum numbers l and m_l,

$$\begin{aligned} L &= \hbar\sqrt{l(l+1)}, \qquad l = 0, 1, 2, \ldots \\ L_z &= \hbar m_l, \qquad m_l = 0, \pm 1, \ldots, \pm l. \end{aligned} \tag{2}$$

What is L and L_z? What can we know about L_x and L_y? What happens when we try to measure L_x? In addition to orbital angular momentum, the electron has also spin angular momentum. What are the relations for electron spin S corresponding to equations (2).

3. The following equation was derived for lattice vibrations

$$M\frac{d^2\xi_n}{dt^2} = K(\xi_{n+1} - 2\xi_n + \xi_{n-1}). \tag{3}$$

Explain the quantities that appear in the equation, and tell from where this equation comes. Solve the equation using the ansatz

$$\xi_n(t) = Ae^{i(kna-\omega t)} \tag{4}$$

and describe the obtained dispersion relation.

4. We study electrons in an infinitely deep one-dimensional potential well, where the electron levels have energies

$$E_n = \frac{\hbar^2\pi^2}{2mL^2}n^2, \qquad n = 1, 2, 3, \ldots \tag{5}$$

Assume the system has four electrons and the width of the potential well $L = 1$ nm.

a) How do the electrons fill the levels in the ground state (corresponding to the temperature $T = 0$)? What is the energy of the lowest empty level?

b) In order to study this system, it is exposed to radiation. The absorption peak corresponding to the lowest excited state is observed at wave length λ_0 of the radiation. Calculate λ_0.

5.

a) What is the essential difference in the filling of the energy bands between insulators, semiconductors and conductors?

b) Describe qualitatively the electron distribution in a metal in the presence of an electric current. What causes electric resistivity of metals?

c) How can you explain that the electric resistivity vanishes in a superconductor?

Collection of formulas

Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t}(\boldsymbol{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\boldsymbol{r},t) + U(\boldsymbol{r})\Psi(\boldsymbol{r},t), \tag{6}$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\boldsymbol{r}) + U(\boldsymbol{r})\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}). \tag{7}$$

Spherical coordinates (r,θ,ϕ):

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}. \tag{8}$$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$, $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$, $\sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha)$

h, Planck's constant, $\hbar = h/2\pi = 1.054 \times 10^{-34}$ Js
$c = 299\,792\,458$ m/s, velocity of light in vacuum
$e = 1.602 \times 10^{-19}$ C, the elementary charge
$k_B = 1.380 \times 10^{-23}$ J/K, Boltzmann constant
$N_A = 6.022 \times 10^{23}$ 1/mol, Avogadro constant
u $= (0.001$ kg/mol$)/N_A = 1.660 \times 10^{-27}$ kg, atomic mass unit
$\epsilon_0 = 8.854 \times 10^{-12}$ C^2/Nm^2, electric constant
$\mu_0 = 4\pi \times 10^{-7}$ N/A^2 $= 4\pi \times 10^{-7}$ T^{2}m^3/J, magnetic constant
$m_e = 9.109 \times 10^{-31}$ kg, mass of an electron
$m_p = 1.6726 \times 10^{-27}$ kg, mass of a proton
1 eV$= 1.602 \times 10^{-19}$ J