Aalto University - School of Science
Department of Mathematics and System Analysis

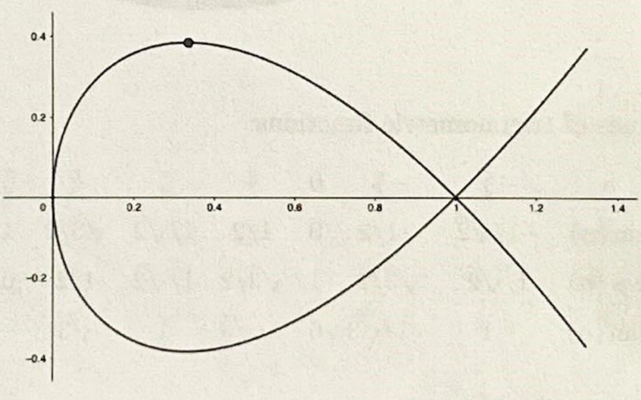
MS-A0211 - Differential and integral calculus 2 Rogovin/Dinesen

Course exam and general exam 19.2.2024 klo 9.00–12.00.

Course exam: the five best problems will be included in the evaluation.
General exam: Do all six problems.
All participants of the lecture course in the period III/2024 can do all six problems. The evaluation is calculated with that option that gives the highest score: either "five best problems + exercise points" or "6 problems only".

No calculators or notes of any kind are allowed.
Each problem is worth 6 points.

1. A plane curve $\mathbf{r}(t) = (t^2, t - t^3)$, resembles the Greek α, when $-1{,}15 \le t \le 1{,}15$ (see the figure).
   (a) Determine the x-coordinate of the "highest point" of the α-curve (marked on the graph). Hint: At the highest point, the tangent vector of the curve is horizontal.
   (b) The curve intersects itself with parameter values $t = \pm 1$. Check whether the intersection is perpendicular, i.e. if the tangent vectors with the two parameter values are perpendicular to each other.
   (c) Form an integral expression for the length of the α-curve. The value of the integral need not be calculated, but in the answer must show the function to be integrated and the limits of the integration.



2. Let $f(x, y) = x^3 + x^2 + 2xy - 3x + y^2$.
   (a) Calculate the Hessian matrix of the function.
   (b) Find the critical points of the function.
   (c) Is the Hessian matrix positive definite, negative definite or indefinite at the critical points?
   (d) Based on the (a)-(c), does the function f have local maximums, local minimums or saddle points?

3. Find the highest and lowest points of the ellipse $x^2 + 3y^2 - 2xy = 6$ by finding the maximum and minimum values of the function $f(x, y) = y$ under the appropriate constraint condition.

4. Does the following functions have a limit at $(0,0)$? If so, what is the limit?

(a) $f(x,y) = \dfrac{2x^4y^2}{x^2+y^2}$

(b) $f(x,y) = \dfrac{-3x^2y}{x^4+y^2}$

5. Evaluate

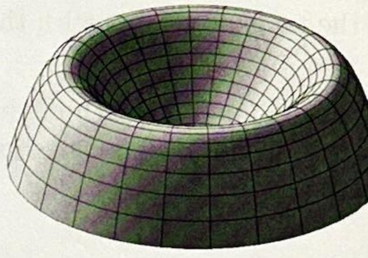
$$I = \iint_T (6x^2 - 3y^2)\, dA$$

over the triangle T with vertices $(0,0)$, $(2,0)$ and $(2,4)$.

6. A millionaire prepares a golden food bowl for his dog. The solid golden part of the bowl B (in the appropriate units) can be described as

$$B = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq z \leq \sin(x^2+y^2) \text{ and } x^2+y^2 \leq \pi\}.$$

Calculate the volume of the solid golden part of the bowl (see the picture).



Extra help: Some values of trigonometric functions:

$$\begin{bmatrix} \alpha & -\frac{\pi}{4} & -\frac{\pi}{6} & 0 & \frac{\pi}{6} & \frac{\pi}{4} & \frac{\pi}{3} & \frac{\pi}{2} & \pi \\ \sin(\alpha) & -1/\sqrt{2} & -1/2 & 0 & 1/2 & 1/\sqrt{2} & \sqrt{3}/2 & 1 & 0 \\ \cos(\alpha) & 1/\sqrt{2} & \sqrt{3}/2 & 1 & \sqrt{3}/2 & 1/\sqrt{2} & 1/2 & 0 & -1 \\ \tan(\alpha) & -1 & -1/\sqrt{3} & 0 & 1/\sqrt{3} & 1 & \sqrt{3} & - & 0 \end{bmatrix}$$

Other formulas without explanation:

$ax^2+bx+c=0 \quad \Rightarrow \quad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$dA = r\, dr d\theta$

$dV = r\, dr d\theta dz$

$dV = r^2 \sin\phi\, dr d\theta d\phi$

Note 1: Answering the course feedback gives one exam point!

Note 2: You can retake the course exam when there is a general exam at April. On the first retake the exercise points of the course are valid. To retake the exam you have to enroll to the exam through Sisu.