Matrix Algebra
MS-A0001
Brzuska/Hakula
Exam, 23.2.2024

Every question carries an equal weight, similarly every part of a question carries an equal weight, unless otherwise specified. There are six problems on this exam. Calculators are not permitted.

PROBLEM 1 A matrix A is called normal, if $A^T A = AA^T$. (a) Show that every orthogonal A is also normal. (b) Under what condition is a real matrix

$$\begin{pmatrix} a & 1 \\ -1 & d \end{pmatrix}$$

normal? Justify your answer.

PROBLEM 2 Let

$$A = \begin{pmatrix} 2 & 1 & 7 \\ 1 & 2 & 8 \\ 2 & 1 & 7 \end{pmatrix}.$$

(a) Show that the columns of A are linearly dependent. (b) Find the nullspace $N(A)$ of A.

PROBLEM 3 Let

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 4 \end{pmatrix}.$$

Compute one possible factorisation $PA = LU$.

PROBLEM 4 Find all solutions to the system $Ax = b$, where

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 8 & 10 \\ 3 & 6 & 11 & 14 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}.$$

PROBLEM 5 . When $a + b = c + d$ show that $x = (1\ 1)^T$ is an eigenvector and find both eigenvalues of

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

PROBLEM 6 What number b in

$$A = \begin{pmatrix} 2 & b \\ 1 & 0 \end{pmatrix}$$

(a) makes it possible to find a diagonal matrix Λ and an orthogonal matrix Q such that $A = Q\Lambda Q^T$? (b) What number makes $A = V\Lambda V^{-1}$ impossible? Justify your answers.