EXAM INFORMATION

Course code: MS-E2135

Course name: Decision Analysis

Exam date: 14.12.2023

Exam duration: Three hours

Teacher: Ahti Salo

Language: English

	Midterm exam	Final exam
Tent type	☐	☑

ALLOWED MATERIAL In the exam, it is only allowed to use a pen, eraser, ruler and compass and the material marked as allowed in the list below. The allowed material is inspected before the exam.

	Allowed	Forbidden
Regular calculator	☑	☐
Graphical calculator (to be emptied)	☑	☐
Formula and table collection	☐	☑
Oher material (listed)	☐	☑

Listed material:

OTHER INSTRUCTIONS

	Yes	No
Exam paper must be returned	☑	☐

Write clearly on every answer sheet the exam date and hall, course code and name, and your study number, name, study programme and signature. Leave material that is not needed in the exam (e.g. mobile phones or other electronic devices) on the sides of the hall or to the supervisors. Keep your student card or identity certificate visible during the exam. Raise your hand to get the supervisor's attention if you need more paper, want to go to the toilet or have questions. When leaving the exam, you must prove your identity and return all answer sheets and scratch papers. You must return at least one answer sheet per exam. Scratch papers are marked with the word "SUTTU" in capital letters and by drawing a large cross over all pages that have writing on them. Scratch papers are not graded. The timetable of the exam is written on the board. The university's exam regulations are followed in the exam.

Exam questions start on the next page. Do not open without permission!

Question 1. Statements (2p)

Which of the following statements are true, and which ones are not? Justify your answers.

(a) The certainty equivalent of an uncertain lottery X will stay the same if the possible outcomes of X are changed so that the resulting revised lottery X' has the same expectation as X, i.e., $E[X'] = E[X]$. (0.5p)

(b) Consider a lottery X with two uncertain outcomes. If the decision-maker's risk premium for this lottery X is equal to zero, then she is risk-neutral overall. (0.5p)

(c) The utopian point in multi-objective optimization is a feasible solution. (0.5p)

(d) In elections with an even number of candidates, it may be that there is no Condorcet winner. (0.5p)

Question 2: Utility theory (3p)

The table below shows four decision alternatives (A,B,C,D) of which all but A have uncertain outcomes. Consider a rational decision-maker (DM) who follows the axioms of utility theory. Out of alternatives A and B, the DM would choose alternative A. Provide justified answers to the following questions (a)-(c).

(a) Does alternative D dominate alternative C in the sense of first-degree stochastic dominance? (1p)

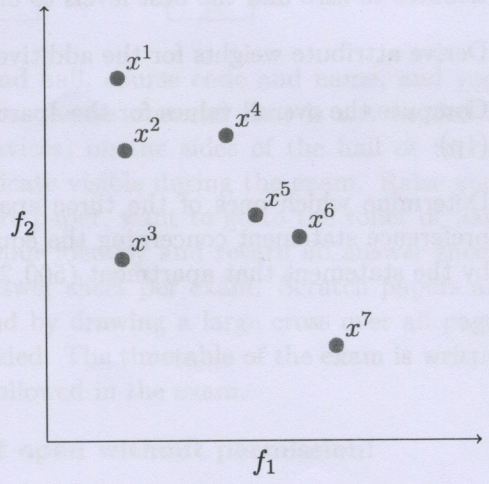
(b) What can you say about the DM's risk attitude? (1p)

(c) Which of the two last alternatives C and D does the DM prefer? (1p)

A		B		C		D	
outcome (€)	probability	outcome (€)	probability	outcome (€)	probability	outcome (€)	probability
70	1	50	0.4	50	0.12	50	0.28
				70	0.7	70	0.3
		100	0.6	100	0.18	100	0.42

Question 3: Multiobjective optimization (2p)

Consider a DM who wishes to maximize two objectives, f_1 and f_2. The values of these objective functions for seven feasible solutions are shown on the right. Values for objective f_1 increase toward the right and those for objective f_2 when moving up.



(a) Which solutions are Pareto-optimal? (1p)

(b) What percentage of the Pareto-optimal solutions can be found using the weighted sum approach? (1p)

Question 4: Decision trees (4p)

You have mineral rights to a plot of land which contains oil with probability 0.10. If there is oil and you choose to drill (this costs 10,000 €), you will receive 200,000 € from oil sales. The alternative is not to drill at all, in which case there are neither costs nor sales revenues. Provide justified answers to the following tasks and questions (a)-(c).

(a) Draw a decision tree to represent your problem. Should you drill? (1p)

(b) What is the Expected Value of Perfect Information? (1p)

(c) Assume that before the drilling decision, you are given the option of consulting a geologist who can assess this piece of land by indicating whether the prospects are "good" or "poor". Yet the geologist is not a perfect predictor. If there is oil, she will say with a probability of 0.95 that the prospects are "good". On the other hand, if there is no oil, the probability of her saying that prospects are "poor" is 0.85. Revise your decision tree to include the "Consult Geologist" option. Calculate the Expected Value of Sample Information for consulting this geologist. Specifically, if she charges 7,000 €, what should you do? (2p)

Question 5: Multi-attribute Value Theory (5p)

Consider a DM who is choosing an office space for her start-up company. The attributes are a_1: *rent (between 400 and 800 €/month)* and a_2: *area (between 25 and 50m^2)*.
Construct an additive multi-attribute value function that is consistent with the preferences (1-3) expressed by the DM:

1. The value of *rent* is decreasing and linear.
2. The value of *area* is increasing, $(35 \leftarrow 25) \sim_d (50 \leftarrow 35)$ and $(45 \leftarrow 35) \sim_d (50 \leftarrow 45)$. You may assume that within the intervals $[25, 35]$, $[35, 45]$ and $[45, 50]$, the value function is linear.
3. Apartments $(400, 25)$ and $(600, 39)$ are equally preferred.

Provide justified answers to the following tasks (a)-(d). Graphical presentation is a sufficient justification for (d).

(a) Construct normalized attribute-specific value functions v_i^N that map the worst levels of attributes to zero and the best levels to one. (2p)

(b) Derive attribute weights for the additive value function $V(x) = \sum_{i=1}^{2} w_i v_i^N(x_i)$. (1p)

(c) Compute the overall values for the apartments, $x^1 = (800, 50)$, $x^2 = (400, 25)$ and $x^3 = (600, 35)$. (1p)

(d) Determine which ones of the three apartments x^1, x^2, and x^3 are non-dominated if the last preference statement concerning the equality of apartments $(400, 25)$ and $(600, 39)$ is replaced by the statement that apartment $(500, 25)$ is preferred to $(800, 50)$? (1p)