

This examination consists of **four questions** (across **three** pages) of which each is graded on a scale from **zero to 25 points**.

Please **justify** your answers and write your **name**, **student registration number**, and **degree programme** clearly on each answer sheet.

Only a **basic calculator** is allowed as extra equipment.

For numerical answers, two decimal are enough.

1 Question I (25 points)

A new potential investor has been reading much news regarding financial marketing and investments. Among the new definitions they have found, the following caught their attention the most:

*The **interest rates** and **inflation** have an intrinsic behaviour. Both are directly related; you cannot express one without considering the other. In addition, all different forms of interest rates (such as **spot**, **forward**, **short**) and financial instruments (**bonds** and **annuities**) should always be considered in light of how inflation behaves in such period.*

Answer the following questions:

1. **Provide** a simple definition for **inflation**, **interest rates** and **bonds**. (9 points).

Please provide your answer within 3 lines of text per definition.

2. **Boostrapping** and **repetition** are two methods to calculate spot rates. Is this statement **correct**? **Justify** your answer. (3 points)

Please provide your answer within 3 lines of text.

3. What is the difference between **perpetual** and **finite length** annuities? (3 points) Please provide your answer within 3 lines of text.

4. Considering an **interest rate** equal to $r = 3\%$ and **inflation** of $f = 1.5\%$. What is the **real interest rate**? (3 points)

5. Considered a **2-year bond** with price $P = 2000$ and an **coupon payment** $C = 500$ and **face value** of $F = 800$. If the spot rate for one year

is **equal** to the real interest (solution from the previous task), what is the **spot rate** for 2 years? (7 points)

2 Question II (25 points)

Suppose the only stocks in the market are those in Table 1. The correlation between the stocks' returns is $\rho_{AB} = 1/2$. There are also risk-free government obligations in the market. The assumptions of the CAPM (Capital Asset Pricing Model) hold and the expected return of the market portfolio is 7.5%.

Table 1: Assets on the market.

Stock	Number of stock	Price	Expected return	Std. of returns
A	100	20€	?	0.15
B	200	15€	0.04	0.10

1. Which is value for **correlaction** between two assets that guarantee that they do not interfere with each other? (2 points)
Please provide your answer within 2 lines of text.
2. What is the **expected return** of stock A? (8 points)
3. What is the **standard deviation** of the market portfolio? (8 points)
4. What are the β 's of the stocks? (7 points)

3 Question III (25 points)

Harry Markowitz is a Nobel Prize Winner economist responsible for a model where the portfolio of an efficient frontier can be found.

Answer the following questions:

1. **Diversification** is the principle where correlated assets are undesirable for an efficient portfolio. Is this statement **correct**? **Justify** this statement. (4 points)
Please provide your answer within 4 lines of text.
2. With random returns, the variance offers a measure of risk, and the expected value highlights the measure of reliability. Is this statement **correct**? (5 points)
Please provide your answer within 5 lines of text.
3. CAPM is an extension of the one-fund theorem where the fund F becomes the market M . How does this affect the investor? Is this model the same as **Markowitz model**? (8 points)
Please provide your answer within 8 lines of text.

4. The most important **parameters** from CAPM (below) is β . What does β_i represent? Is β a **useful** guide for investment selection? **Justify** your answer. (8 points)
Please provide your answer within 8 lines of text.

4 Question IV (25 points)

A stock is currently valued at $S(0) = 50 \text{ €}$, and its relative monthly price changes up and down are described by the parameters $u = 1.20$ and $d = 0.83$, respectively. The price of this stock moves upward each month with probability $p = 0.65$. The annual risk-free rate is $r_f = 5\%$.

1. Based on a **binomial lattice**, what is the price of an **American put option** on this stock, assuming that this option expires in **three months** with a strike price K of 65 €? (12 points)
2. With what **probabilities** are each of the **end states** of this **binomial lattice** reached? (10 points)
3. The **binomial lattice** used in previous questions is an **additive** model. Is this sentence **correct**? Justify your answer. (3 points)
Please provide your answer within 2 lines of text.

A Useful Equations

Real interest rate:

$$1 + r_0 = \frac{1 + r}{1 + f}$$

Spot rates:

$$P = \frac{C}{1 + s_1} + \frac{C + F}{(1 + s_2)^2}$$

Forward rates (yearly compound-
ing):

$$f_{ij} = \left(\frac{(1 + s_j)^j}{(1 + s_i)^i} \right)^{\frac{1}{j-i}} - 1$$

Expected values:

$$r_m = w_a r_a + w_b r_b$$

Covariance:

$$\sigma_m^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} = w^T \Sigma w$$

where:

$$w = [w_1, \dots, w_n] \text{ and } \Sigma =$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_n^2 \end{bmatrix}$$

$$\text{cov}(r_A, r_m) = w_A \sigma_A^2 + w_B \rho_{AB} \sigma_A \sigma_B$$

Beta:

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2}$$

Options:

$$R = 1 + \frac{r_f}{12}$$

$$P = \max\left\{\frac{1}{R}(qP_u + (1 - q)P_d), K - S\right\}$$

$$P = \max\{K - S, 0\}$$

2. Question II (25 points)

Suppose the only stocks in the market are those in Table 1. The annual risk-free rate is $r = 5\%$. The price of this stock moves upward each month with probability $q = 0.65$. The price of this stock moves downward each month with probability $1 - q = 0.35$. The price of this stock is currently $P = 100$. The price of this stock is currently $P = 100$. The price of this stock is currently $P = 100$.

Stock	Number of Shares	Current Price
A	100	100
B	100	100

1. Which two probabilities are each of the end states of this binomial lattice tree? (2 points)
2. What is the expected return on the market portfolio? (2 points)
3. What is the standard deviation of the market portfolio? (2 points)
4. What are the expected returns on the two stocks? (2 points)

3. Question III (25 points)

Harry Markowitz is a pioneer in portfolio theory. He showed that the efficient frontier of risky assets is a curve in the risk-return plane. The efficient frontier is the set of portfolios that offer the highest expected return for a given level of risk. The efficient frontier is the set of portfolios that offer the highest expected return for a given level of risk.

1. Diversification is a key concept in portfolio theory. It refers to the process of spreading investments across different assets to reduce risk. Diversification is a key concept in portfolio theory. It refers to the process of spreading investments across different assets to reduce risk.
2. With a risk-averse utility function, the optimal portfolio is the one that maximizes expected utility. With a risk-averse utility function, the optimal portfolio is the one that maximizes expected utility.
3. The Capital Asset Pricing Model (CAPM) is a framework for determining the expected return on an asset. The Capital Asset Pricing Model (CAPM) is a framework for determining the expected return on an asset.