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Department of Computer Science
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CS-C2160 Theory of Computation (5 cr)
Course Exam 07 June 2024, 09:00–12:00

Instructions

You are allowed to bring with you a single two-sided A4 "cheat sheet", **personally handwritten by you. (NO photocopies, NO printouts, NO computer type-set text.)** Please include your name and student ID at the top of the cheat sheet, and return it together with your answer sheets at the end of the exam.
Note: If you have not completed your online A+ home assignments, your exam will not be graded.

Problems

1. Let L be the language described by the regular expression $(ab \cup aba)^*$.

   (a) Design a nondeterministic finite state automaton (without ε-transitions) that recognises L.

   (b) Determine the minimal deterministic finite state automaton that recognises L.

   (7 + 8 pts)

2. (a) Design a context-free grammar for the language

   $$L = \{\, [^i 1^j 0^k]^i \mid i, j, k \geq 0,\ j \geq k \}.$$

   Draw the corresponding parse trees for the words [[1]] and 110.

   (b) Prove (precisely!) that the language L discussed in part (a) is not regular.

   (8+7 pts)

3. (a) Prove that if the languages $L \subseteq \{0, 1, \#\}^*$ and $L' \subseteq \{0, 1\}^*$ are context-free, then so is the language $L'' = L[L'] \subseteq \{0, 1\}^*$, whose words are obtained from the words in L by replacing each #-symbol by some word in L' (not necessarily always the same).

   (b) Is the claim true also for regular languages, i.e. if $L \subseteq \{0, 1, \#\}^*$ and $L' \subseteq \{0, 1\}^*$ are regular, then so is $L'' = L[L'] \subseteq \{0, 1\}^*$? Justify your answer. (7 + 8 pts)

4. Which of the following claims are true and which are false? Provide a brief justification for each of your answers, based on results introduced at the course. Note that just stating "True" or "False" without justification will not earn you any points. (For example if the claim was: "The complement of any decidable language is semidecidable", your answer could be: "True. The complement of any decidable language is decidable (by switching the accepting and rejecting states in the recognising TM), and all decidable languages are by definition also semidecidable.")

   (a) Deterministic pushdown automata can recognise some nonregular languages.

(b) Every decidable language can be generated by a context-free grammar.

(c) Any context-free language can be recognised by a deterministic Turing machine.

(d) The halting problem for Turing machines is semidecidable.

(e) The complement of any semidecidable language is also semidecidable. (5 · 3 pts)