Aalto University School of Science
Mat-1.1632 Mathematics 3-II
Alestalo

Exam 12.8.2013.
Please fill in all the required information to each exam paper.
Calculators are allowed.

1. Find the LU-decomposition of the matrix
$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & 0 \end{bmatrix}.$$

2. Show that the equation
$$3x^2 + 4xy + 6y^2 = 14$$
presents an ellipse and find the lengths of its semi-axes.

3. The eigenpairs (of the form (eigenvalue, corresponding eigenvector)) of the matrix
$$A = \begin{bmatrix} 7 & 9 & -5 \\ -5 & -7 & 5 \\ -1 & -1 & 3 \end{bmatrix}$$
are $(2, [1,0,1]^T)$, $(-2, [-1,1,0]^T)$ and $(3, \mathbf{v_3})$.
a) Find the eigenvector $\mathbf{v_3}$.
b) Solve the initial value problem $\mathbf{y}' = A\mathbf{y}$, $\mathbf{y}(0) = [0,0,7]^T$.

4. Consider the non-linear system
$$\begin{cases} y_1' = 2y_2 - 2y_1y_2 \\ y_2' = y_1 - y_2. \end{cases}$$
a) What are the critical points of the system?
b) Linearize the system at every critical point and determine their stability.
c) Using the Euler method with one step, calculate an approximation to $\mathbf{y}(0.1)$, when $\mathbf{y}(0) = [1,-2]^T$.

5. The general solution to the wave equation $u_{tt} = c^2 u_{xx}$ in the domain $0 < x < \pi,\ t > 0$, is given by
$$u(x,t) = \sum_{n=1}^{\infty} (A_n \cos(cnt) + B_n \sin(cnt)) \sin(nx)$$
if $u(0,t) = u(\pi,t) = 0$ for all $t \geq 0$.
a) Find the solution corresponding to the initial conditions $u(x,0) = 3\sin x - 4\sin(5x)$ and $u_t(x,0) = 3\sin(2x)$.
b) Derive formulas for A_n and B_n corresponding to the initial conditions $u(x,0) = f(x)$ and $u_t(x,0) = 0$.