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CS-C2160 Theory of Computation (5 cr)
Course Exam 16 October 2024, 13:00–16:00

Instructions

You are allowed to bring with you a single two-sided A4 "cheat sheet", **personally handwritten by you. (NO photocopies, NO printouts, NO computer type-set text.)** Please include your name and student ID at the top of the cheat sheet, and return it together with your answer sheets at the end of the exam.
Note: If you have not completed your online A+ home assignments, your exam will not be graded.

Problems

1. Design **deterministic** finite automata for recognising the following languages:

   (a) $\{w \in \{a,b\}^* \mid w$ contains the substring bab (as consecutive symbols)$\}$,

   (b) $\{w \in \{a,b,c\}^* \mid w$ has at most 3 symbols that are b or $c\}$,

   (c) $\{w \in \{a,b,c\}^* \mid bacc$ appears as a subsequence of w (not necessarily consecutively)$\}$.

   Give **regular expressions** that describe the following languages:

   (d) $\{w = a_1a_2,\ldots,a_{2n} \in \{0,1\}^* \mid n \in \mathbb{N}$, and (all 1 symbols of w are in even positions or all 1 symbols of w are in odd positions)$\}$

   (e) $\{w \in \{0,1\}^* \mid$ **if** w ends with 00 **then** it starts with 11$\}$. 15 points

2. For a string $w \in \{a,b,c\}^*$, let us denote by $n_s(w)$ the number of s symbols in w. Design context-free grammars for the following languages:

   (a) $L_{=2} = \{ww' \mid w,w' \in \{a,b\}^*$ and $n_a(w) = n_b(w') = 2\}$, (3pts)

   (b) $L_+ = \{w \in \{a,b,c\}^* \mid n_a(w) + n_b(w) = 2n_c(w)\}$, (3pts)

   Hint: In part (b), the symbols a and b play the same role.

   Give parse trees in your grammars for the following strings:

   (c) $ababbab \in L_{=2}$, (2pts)

   (d) $cabbac \in L_+$. (2pts)

   Finally,

   (e) Prove that L_+ is not regular. (5pts) 15 points

3. *Choose one* of the following:

(a) Show that if a language L is context-free then so are the following languages:

$$L^* = \bigcup_{k \geq 0} L^k = \{w_1 \dots w_k \mid k \geq 0,\ w_i \in L \text{ for all } i = 1, \dots, k\},$$

$$L^R = \{w^R \mid w \in L\}.$$

(Here w^R denotes the *reversal* of string w, i.e. w written backwards.

(b) Show that if languages L_1 and L_2 are semidecidable, then so are the languages $L_1 \cap L_2$ and $L_1 L_2 = \{w_1 w_2 \mid w_1 \in L_1, w_2 \in L_2\}$.

15 points

4. Which of the following claims are true and which are false? Provide a brief justification for each of your answers, based on results introduced at the course. Note that just stating "True" or "False" without justification will not earn you any points. (For example if the claim was: "The complement of any decidable language is semidecidable", your answer could be: "True. The complement of any decidable language is decidable (by switching the accepting and rejecting states in the recognising TM), and all decidable languages are by definition also semidecidable.")

(a) The intersection of two regular languages is not always context-free.

(b) The problem of determining if two nondeterministic finite automata are equivalent, i.e. recognise the same language, is decidable.

(c) The problem of determining if a given string is *not* generated by a context-free grammar is decidable.

(d) The problem of determining if a Turing machines halts on some input is semidecidable.

(e) There are undecidable languages A, B such that $A \cap B = \emptyset$ and $A \cup B = \{0,1\}^*$ 15 points