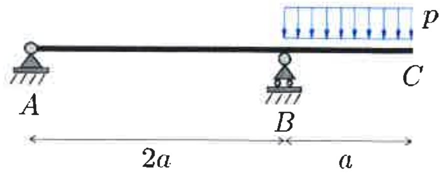
CIV-E1020 Mechanics of Beam and Frame Structures

Examination 7.12.2023

There are only three (3) compulsory problems to solve.

1. [5p] Given a beam of length $3a$ with a simple fixed support in *A* and a simple support permitting horizontal motion in *B*. The span *AB* has length $2a$. The span *BC* has length a. The beam is linear elastic with constant bending stiffness EI, constant shear stiffness GA_z, and constant extension stiffness EA. The beam is loaded by a uniformly distributed load with intensity p acting on the span *BC*.

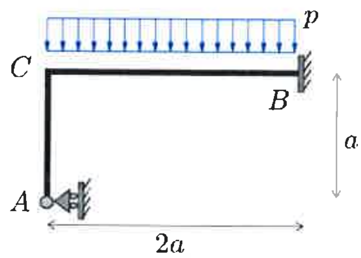


a) Determine the support reactions. Hint: Use balance equations for that.

b) Determine and draw the distributions of the bending moment, shear and axial forces.

c) Use the principle of virtual work to determine the rotation θ_C at the tip *C* of the beam. In the equation of virtual work, account for effects of bending moment, shear and axial forces.

2. [5p] Given an angle frame with a fixed support in *B* and a simple support permitting vertical motion in *A*. A vertical uniformly distributed load with intensity p is acting on the beam *CB*. The beam *AC* has length a. The beam *CB* has length $2a$. The beams are linear elastic with constant bending stiffness EI.

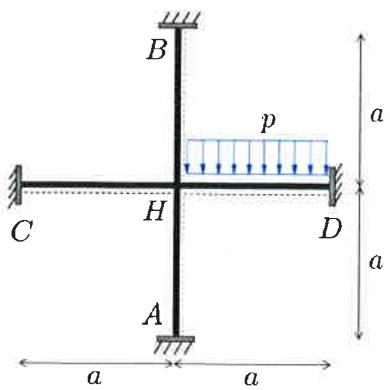


a) Use the force method to determine the internal moment in the joint *C*. Account only for effect of bending moment.

b) Determine the reactions.

c) Determine and draw the distributions of the bending moment, shear and axial forces.

3. [5p] Given a cross frame with fixed supports in *A, B, C* and *D*. The beams *AH, CH, HB,* and *HD* have length a. A uniformly distributed load with intensity p is acting in the vertical direction on the beam *HD*. The beams are linear elastic with constant bending stiffness EI.



a) Use Slope-Deflection Method and determine the bending moment at clamping support *D*.

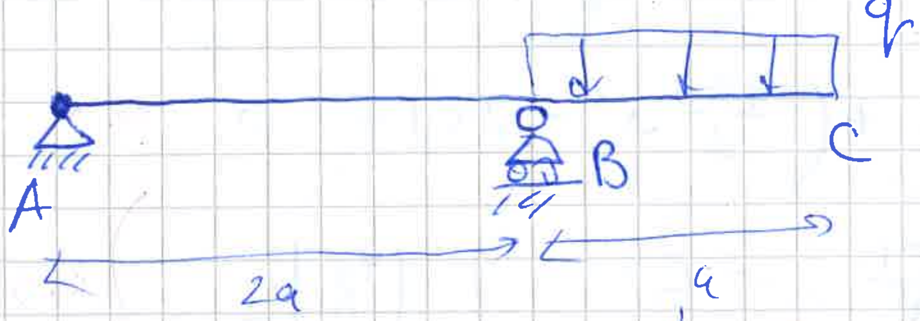
b) Draw accurately the bending moment diagram.

Hint: Determine the rest bending moments: at clamping supports *A*, *B*, and *C* as well as at joint *H*. Remember the sign convention for the bending moment.

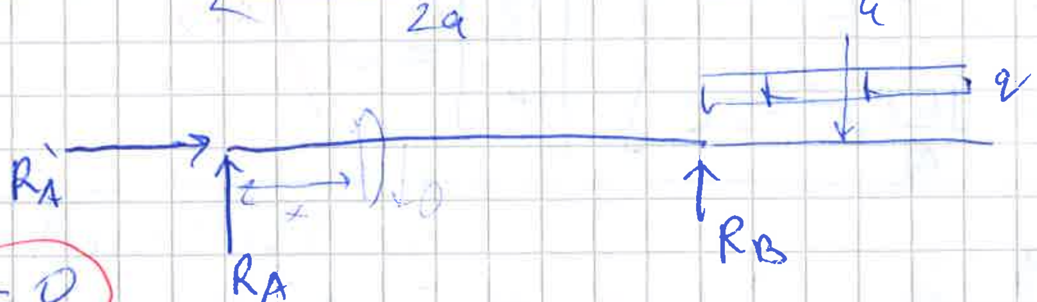
Exam 2. 7.12.2023

(1).

Case 0:



a).

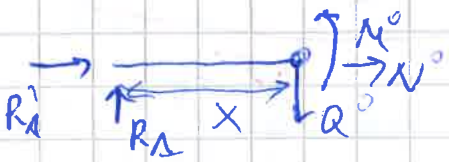


$R_A' = 0$

B) $-R_A 2a - q\frac{a^2}{2} = 0 \Rightarrow R_A = -\frac{qa}{4}$

↑: $R_A + R_B - qa = 0 \Rightarrow R_B = \frac{5}{4}qa$ (1.)

b). AB:

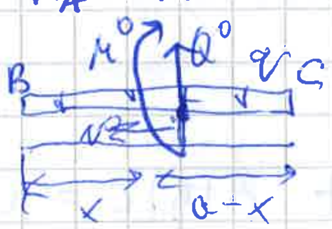


→: $N^0 = 0$

↑: $Q^0 = -\frac{qa}{4}$

x): $-R_A x + M^0 = 0 \Rightarrow M^0 = -\frac{qa}{4}x$

BC:



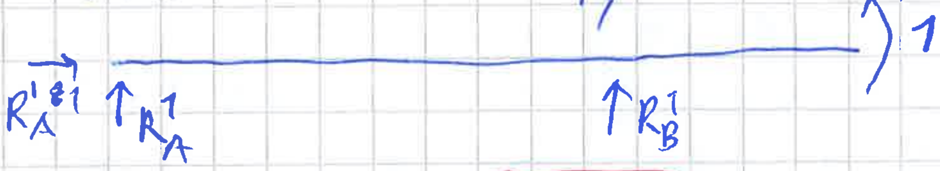
→: $N^0 = 0$

↑: $Q^0 = q(a-x)$

x): $-M^0 - q\frac{(a-x)^2}{2} = 0 \Rightarrow M^0 = -\frac{q}{2}(x-a)^2$

Case 1:

c).



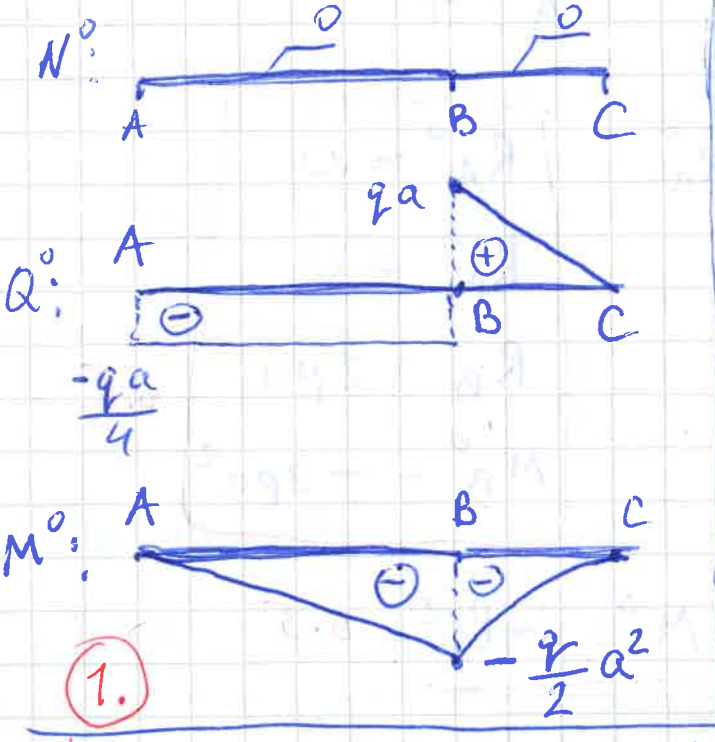
→: $R_A'^1 = 0$

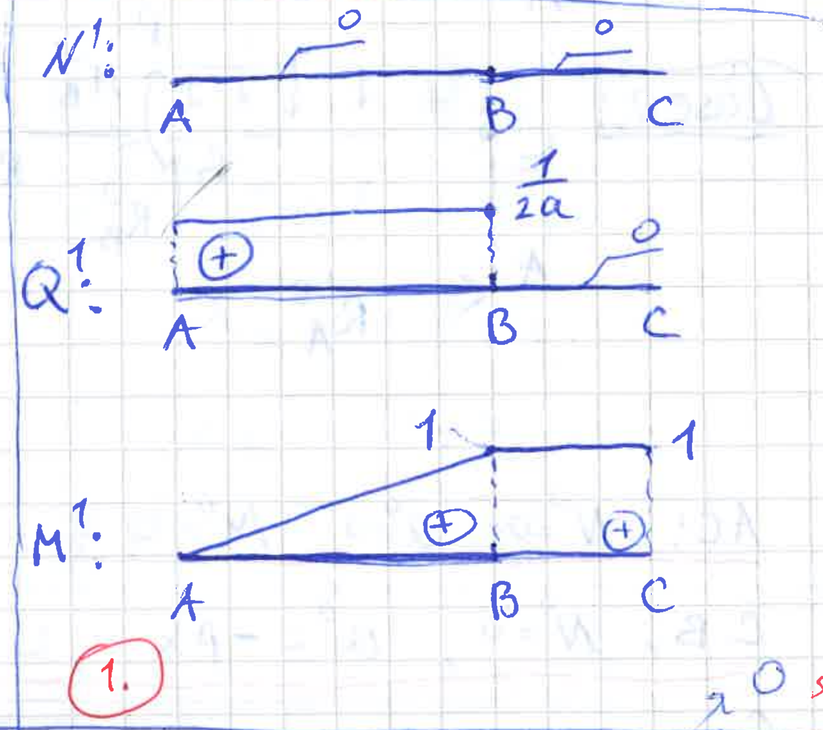
B) $-R_A^1 2a + 1 = 0 \Rightarrow R_A^1 = \frac{1}{2a}$

↑: $R_A^1 + R_B^1 = 0 \Rightarrow R_B^1 = -\frac{1}{2a}$

AB: $N^1 = 0$, $Q^1 = \frac{1}{2a}$, $M^1 = \frac{x}{2a}$

BC: $N^1 = 0$, $Q^1 = 0$, $M^1 = 1$





0.5 PVW: $\theta_c = \int_{ABC} \frac{M^0 M^1}{EJ} ds + \int_{ABC} \frac{Q^0 Q^1}{GA_z} ds + \cancel{\int_{ABC} \frac{N^0 N^1}{AE} ds}$ → 0 0.5

$$\int_{ABC} \frac{M^0 M^1}{EJ} ds = \frac{1}{EJ} \int_0^{2a} \left(-\frac{qa}{4} x\right) \frac{x}{2a} dx + \frac{1}{EJ} \int_0^a \left[-\frac{q}{2}(x-a)^2\right] dx$$

$$= -\frac{q}{8EJ} \int_0^{2a} x^2 dx - \frac{q}{2EJ} \int_0^a (x-a)^2 dx$$

$$= -\frac{q}{8EJ} \left.\frac{x^3}{3}\right|_0^{2a} - \frac{q}{2EJ} \left.\frac{(x-a)^3}{3}\right|_0^a =$$

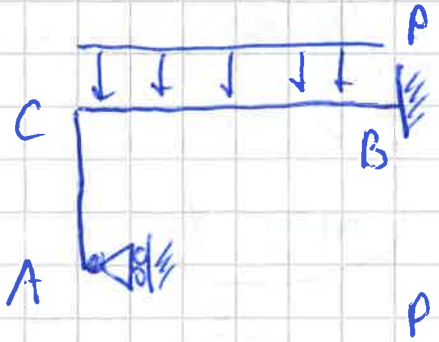
$$= -\frac{qa^3}{3EJ} - \frac{qa^3}{6EJ} = -\frac{qa^3}{2EJ}$$

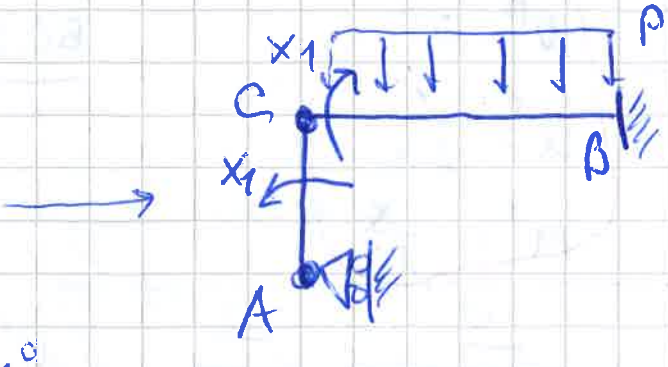
$$\int_{ABC} \frac{Q^0 Q^1}{GA_z} ds = \int_0^{2a} \frac{1}{GA_z} \left(-\frac{qa}{4}\right)\left(\frac{1}{2a}\right) dx = -\frac{qa}{4GA_z}$$

$$\boxed{\theta_c = \underbrace{-\frac{qa^3}{2EJ}}_{1.} \underbrace{- \frac{qa}{4GA_z}}_{0.5}}$$ 1.5

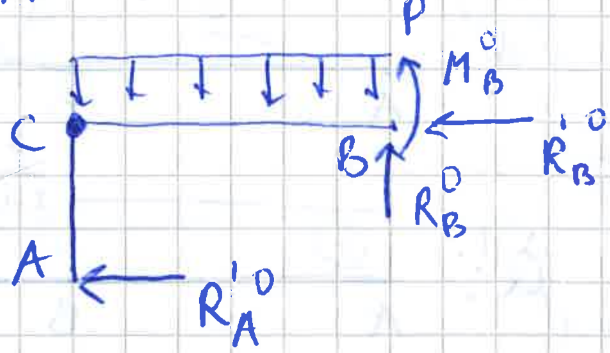
2

a)





Case 0.



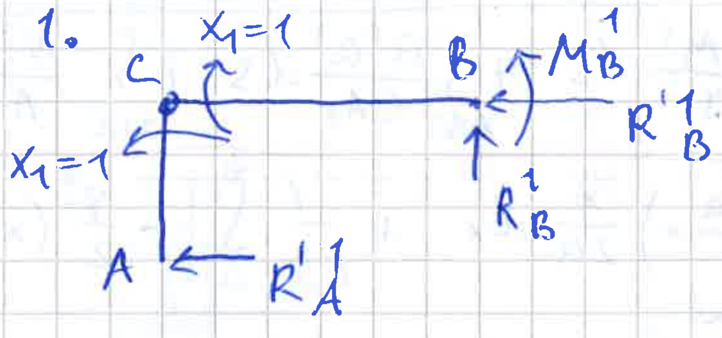
$$R_A'^0 = 0$$
$$R_B'^0 = 0$$
$$R_B^0 = 2pa$$
$$M_B^0 = -2pa^2$$

0.5

AC: $N^0 = 0$, $Q^0 = 0$, $M^0 = 0$

CB: $N^0 = 0$, $Q^0 = -px$, $M^0 = -\frac{px^2}{2}$ 0.5

Case 1.



$C\circlearrowleft: X_1 - R_A'^1 a = 0$

$R_A'^1 = \frac{1}{a}$

$\rightarrow: R_A'^1 + R_B'^1 = 0 \Rightarrow R_B'^1 = -\frac{1}{a}$

$\uparrow: R_B^1 = 0$

$C\circlearrowleft: -X_1 + M_B^1 = 0 \Rightarrow M_B^1 = 1$

0.5

AC: $N^1 = 0$, $Q^1 = \frac{1}{a}$, $M^1 = \frac{x}{a}$ 0.5

CB: $N^1 = \frac{1}{a}$, $Q^1 = 0$, $M^1 = 1$

$$\delta_{10} = \int_{ACB} \frac{M^0 M^1}{EJ} ds = \int_{AC} \cancel{\frac{M^0 M^1}{EJ}}^{\,0} ds + \int_{CB} \frac{M^0 M^1}{EJ} ds$$

$$= \frac{1}{EJ} \int_0^{2a} \left(-\frac{px^2}{2}\right) dx = -\frac{p}{2EJ} \left.\frac{x^3}{3}\right|_0^{2a} = -\frac{4pa^3}{3EJ}$$ 0.5

$$\delta_{11} = \int_{ACB} \frac{M^1 M^1}{EJ} ds = \frac{1}{EJ}\int_0^a \left(\frac{x}{a}\right)^2 dx + \frac{1}{EJ}\int_0^{2a} dx$$

$$= \frac{1}{EJa^2} \left.\frac{x^3}{3}\right|_0^a + \frac{1}{EJ} \left. x \right|_0^{2a} = \frac{7}{3}\frac{a}{EJ}$$ 0.5

0.5

$$\delta_1 = \delta_{10} + \delta_{11} X_1 = 0 \quad \Rightarrow \quad X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{4}{7} pa^2 = M_c$$

b). $R'_A = R'^{0}_A + R'^{1}_A X_1 = 0 + \frac{1}{a}\cdot\frac{4}{7}pa^2 = \frac{4}{7}pa$

$R'_A + R'_B = 0 \quad \Rightarrow \quad R'_B = -\frac{4}{7}pa$ 0.5

$R_B = 2pa$

$M_B = M_B^0 + M_B^1 X_1 = -2pa^2 + \frac{4}{7}pa^2 = -\frac{10}{7}pa^2$

c). AC: $N = N^0 + N^1 X_1 = 0$

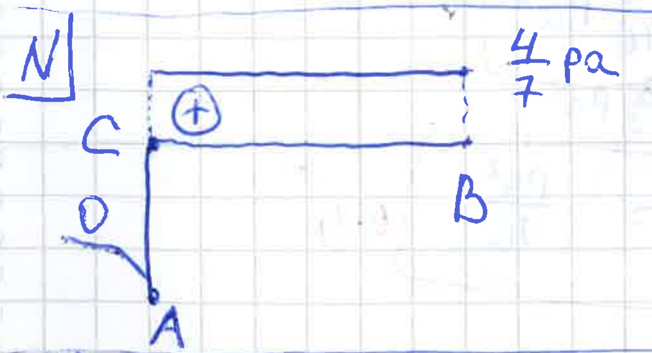
$Q = Q^0 + Q^1 X_1 = \frac{4}{7}pa$

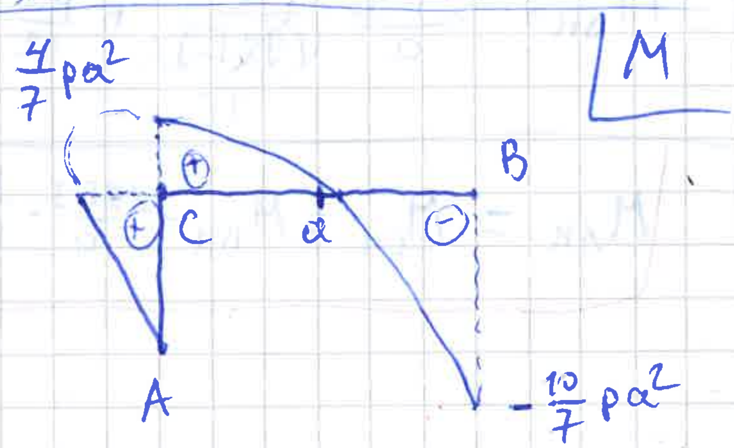
$M = M^0 + M^1 X_1 = \frac{4}{7}pa\,x$

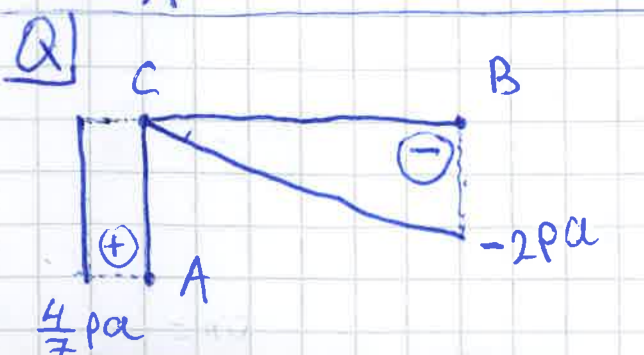
CB: $N = \frac{4}{7}pa$

$Q = -px$

$M = \frac{p}{2}\left(-x^2 + \frac{8}{7}a^2\right)$

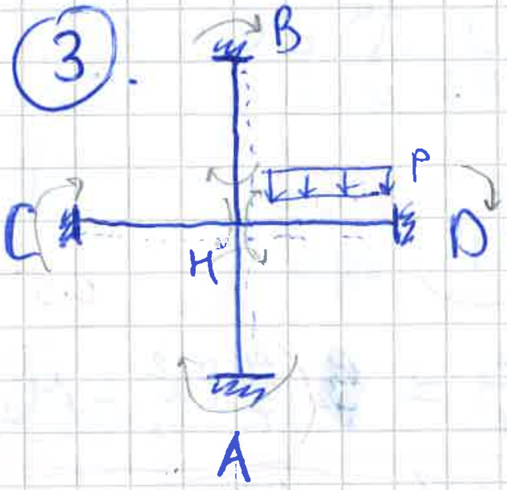






1.

(3).



$\boxed{n_\psi = 0, \quad n_\varphi = 1}$ 0.5

$\varphi_{AH} = \varphi_{BH} = \varphi_{CH} = \varphi_{DH} = 0$ 0.5

$\varphi_{HA} = \varphi_{HC} = \varphi_{HB} = \varphi_{HD} = \varphi$ 0.5

$M_{HA} + M_{HC} + M_{HB} + M_{HD} = 0$

$$M_{ij} = a_{ij}\varphi_{ij} + b_{ij}\varphi_{ji} - c_{ij}\psi_{ij} + \bar{M}_{ij}$$

$a_{ij} = \frac{4EJ}{a}$

$b_{ij} = \frac{2EJ}{a}$

$\bar{M}_{HD} = -\frac{Pa^2}{12}$

$\bar{M}_{DH} = \frac{Pa^2}{12}$

0.5

$M_{HA} = a_{HA}\varphi = \frac{Pa^2}{48}$

$M_{HC} = a_{HC}\varphi = \frac{Pa^2}{48}$

$M_{HB} = a_{HB}\varphi = \frac{Pa^2}{48}$

$M_{HD} = a_{HD}\varphi - \frac{Pa^2}{12} = -\frac{Pa^2}{16}$

0.5

$$(a_{HA} + a_{HC} + a_{HB} + a_{HD})\varphi - \frac{Pa^2}{12} = 0$$

$$4 \cdot \frac{4EJ}{a}\varphi - \frac{Pa^2}{12} = 0 \Rightarrow \boxed{\varphi = \frac{Pa^3}{192EJ}}$$ 0.5

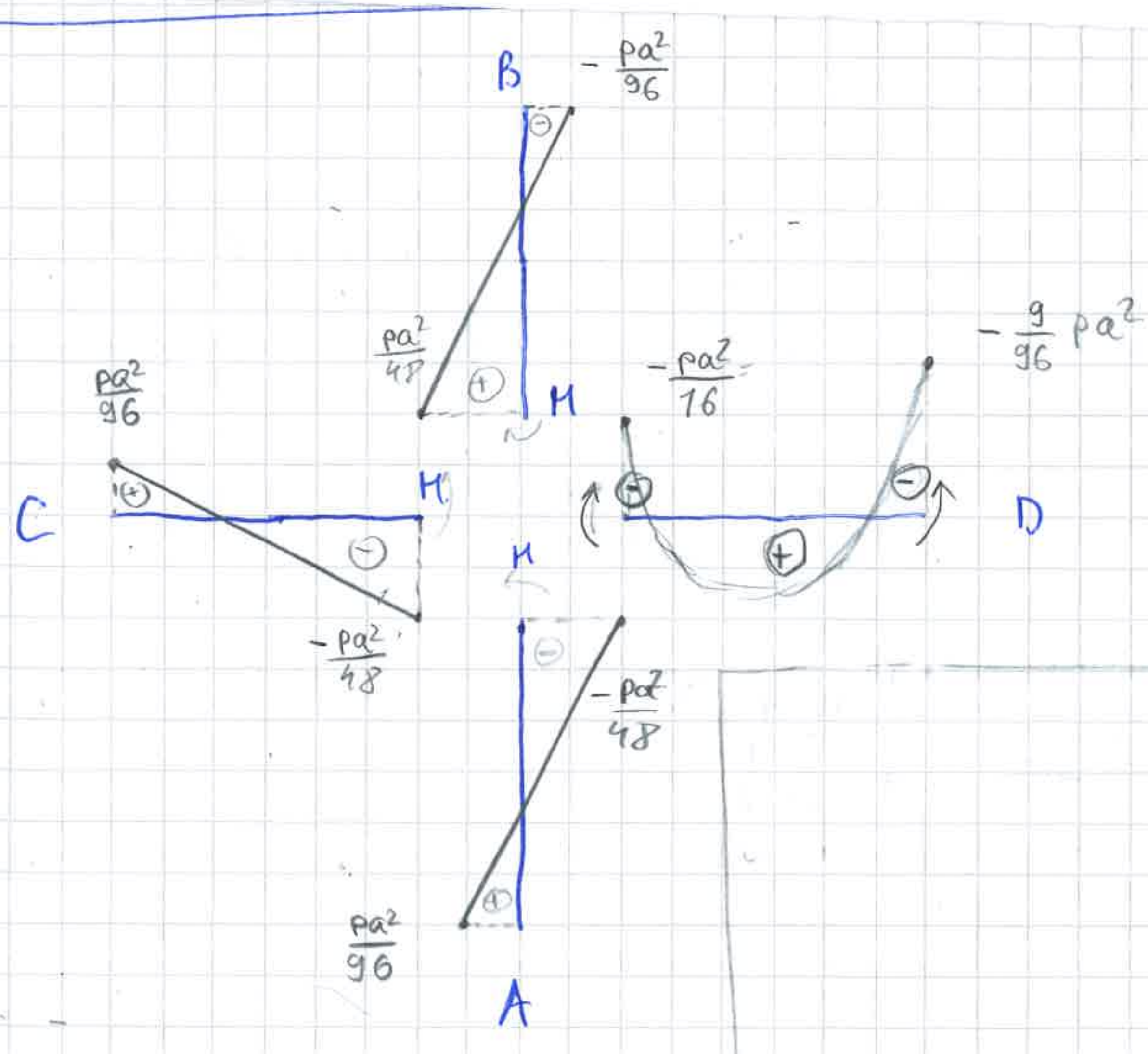
$$M_{DH} = a_{DH}\cancelto{0}{\varphi_{DH}} + b_{DH}\varphi_{HD} + \bar{M}_{DH} - c_{DH}\cancelto{0}{\psi_{DH}}$$ 0.5

$$M_{DH} = \frac{2EJ}{a} \cdot \frac{Pa^3}{192EJ_{96}} + \frac{Pa^2}{12} = \boxed{\frac{9}{96}Pa^2}$$

$\frac{3}{32}Pa^2$

$$\boxed{M_{AH} = M_{CH} = M_{BH} = \frac{2EJ}{a} \cdot \frac{Pa^3}{192EJ} = \frac{Pa^2}{96}}$$ 0.5

1



$$M): \; -\frac{pa^2}{48} - \frac{pa^2}{48} + \frac{pa^2}{16} - \frac{pa^2}{48} = 0$$