[December 2024] **CHEM-E7190 (Exam)**

Exercise 01. After the linearisation of a process dynamics and instrument model around some steady-state operating point (x_{SS}, u_{SS}), we have the following linear and time-invariant (LTI) dynamics and measurement equations:

$$\dot{x}(t) = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u(t) \tag{1a}$$

$$y(t) = \begin{bmatrix} 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \end{bmatrix} u(t) \tag{1b}$$

- (10%) Briefly define what is the meaning of Eq. (1a) and Eq. (1b). For each equation, use simple words and max two short sentences;
- (10%) Briefly define what the process variables $\dot{x}(t)$, $x(t)$, $u(t)$, and $y(t)$ are and determine how many (their dimension) of each are used in Eq. (1). For each variable, use simple words and max two short sentences;
- (10%) Identify all the matrices involved in the model (state-state, input-state, state-output, and input-output matrix) and determine their size;
- (5%) Define the notion of stability for LTI system models. Use simple words and max two short sentences;
- (5%) Determine whether the process model in Eq. (1) is stable. Comment on the procedure you used and the conclusion you reached.

Exercise 02. We are interested in controlling the process above using a state-feedback approach. We must verify whether the model is controllable and observable and then design a state feedback controller and a state observer.

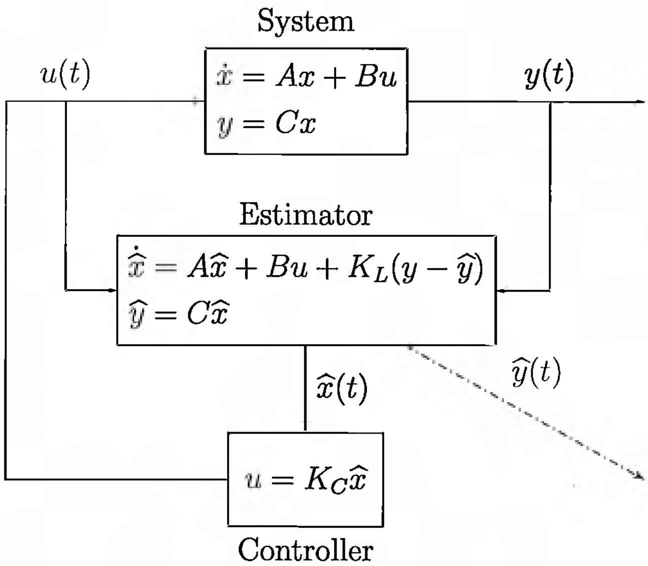
- (5%) Define the notion of controllability and how a state feedback controller operates. Use simple words and max three short sentences;
- (5%) Establish, graphically, whether the system is likely to be controllable, then comment on the procedure you used and the conclusion you reached;
- (10%) Define the controllability matrix and compute it for the model in Eq. (1). Determine its controllability using the controllability matrix and comment on whether this conclusions matches the graphical result.

(5%) Define the notion of observability and how a state observer operates. Use simple words and max three short sentences;

(5%) Establish, graphically, whether the system is likely to be observable, then comment on the procedure you used and the conclusion you reached;

(10%) Define the observability matrix and compute it for the model in Eq. (1). Determine its observability using the observability matrix and comment on whether this conclusions matches the graphical result.

The block-diagram representing the system-observer-controller is below



The observer is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_L(y(t) - \hat{y}(t)); \tag{2a}$$
$$\hat{y}(t) = C\hat{x}(t). \tag{2b}$$

The controller is given by

$$u(t) = K_C\hat{x}(t). \tag{3}$$

(10%) Briefly define what is the meaning of Eq. (2a), Eq. (2b), and Eq. (3). For each equation, use simple words and max six short sentences;

(10%) Briefly define what the process variables $\dot{\hat{x}}(t)$, $\hat{y}(t)$ are and determine how many (their dimension) of each are used in process control based on model in Eq. (1). Use simple words and max four short sentences.