1. We study electrons in an infinitely deep one-dimensional potential well, whose width is L.
   a) Solve the time-independent Schrödinger equation. Write the normalized wave functions and energies of the energy levels.
   b) Assume the system has four electrons, and we neglect their mutual interaction. How do the electrons fill the levels in the ground state (corresponding to the temperature $T = 0$)? What is the the lowest energy to excite the system?

2.
   a) Explain what is a Bravais lattice.
   b) Explain the two close packed structures. How do they differ from each other?
   c) How can we explain the stability of structures like diamond and sodium chloride, which are not close-packed structures?

3. The speed of longitudinal sound in iron is 5500 m/s. Based on this estimate the maximal frequency of lattice vibrations assuming the nearest neighbor distance of iron atoms is 250 pm. Estimate the temperature below which the heat capacity of iron is reduced from its value based on classical mechanics. Hint: in the course the following dispersion relation was derived for lattice vibrations.

$$\omega = 2\sqrt{\frac{K}{M}} \sin\frac{ka}{2}. \quad (1)$$

4. In Drude model one writes the equation

$$\frac{d\boldsymbol{p}}{dt} = -\frac{\boldsymbol{p}}{\tau} - e\boldsymbol{\mathcal{E}}. \quad (2)$$

   a) Define the quantities that appear in the equation and explain the physics what each term describes.
   b) The electric current density is obtained from the formula $\boldsymbol{j} = -en_e\boldsymbol{v}$ where $\boldsymbol{v}$ is the average velocity of the conduction electrons and n_e is their number density. Derive from this information the Drude equation for the electrical conductivity.
   c) What essential differences between metals and semiconductors one has to take into account in order to apply the Drude model to semiconductors?

5. Explain with words and pictures how a junction between n and p type semiconductors acts as a rectifier of electric current.

Collection of formulas

Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t}(\boldsymbol{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\boldsymbol{r},t) + U(\boldsymbol{r})\Psi(\boldsymbol{r},t), \tag{3}$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\boldsymbol{r}) + U(\boldsymbol{r})\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r}). \tag{4}$$

Spherical coordinates (r, θ, ϕ):

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}. \tag{5}$$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\sin 2\alpha = 2\sin\alpha\cos\alpha$, $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
$\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$, $\sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha)$

h, Planck's constant, $\hbar = h/2\pi = 1.054 \times 10^{-34}$ Js
$c = 299\,792\,458\,\mathrm{m/s}$, velocity of light in vacuum
$e = 1.602 \times 10^{-19}$ C, the elementary charge
$k_\mathrm{B} = 1.380 \times 10^{-23}$ J/K, Boltzmann constant
$N_\mathrm{A} = 6.022 \times 10^{23}$ 1/mol, Avogadro constant
$\mathrm{u} = (0.001\ \mathrm{kg/mol})/N_\mathrm{A} = 1.660 \times 10^{-27}$ kg, atomic mass unit
$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/Nm^2}$, electric constant
$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2} = 4\pi \times 10^{-7}\ \mathrm{T^2m^3/J}$, magnetic constant
$m_e = 9.109 \times 10^{-31}$ kg, mass of an electron
$m_p = 1.6726 \times 10^{-27}$ kg, mass of a proton
1 eV$= 1.602 \times 10^{-19}$ J