EXAM,
MS-A0409 FOUNDATIONS OF DISCRETE MATHEMATICS
7. JUNE 2024

- Time: 3 hours
- Equipment: Writing equipment and one page (A4) of handwritten notes. The notes should be marked with the student's name, but do not have to be handed in.
- Your name and student number should be clearly visible on every answer sheet.
- Answer all questions to the best of your ability. Every question is worth 4 points.
- Motivate your solutions carefully. Answers without motivations give no points.

Problem 1

For each of the following composed statements, determine whether or not it is a tautology.

(a) $(P \to (P \land Q)) \to (P \to Q)$
(b) $((P \to Q) \to Q) \to (P \lor Q)$
(c) $((P \land \neg P) \to Q) \to ((P \lor \neg P) \to R)$
(d) $(Q \to (P \land \neg P)) \to (R \to (P \land \neg P))$

Problem 2

Prove using induction that $2^{n+2} + 3^{2n+1}$ is divisible by 7 for all $n \in \mathbb{N}$.

Problem 3

During an exam period on a small college, seven exams are organized: architecture, biochemistry, calculus, discrete mathematics, electrotechnology, fysics(!) and geography. Some students want to write more than one of the exams, wherefore the college wishes to schedule the exams as follows:

- Architecture should not be written on the same day as neither biochemistry, calculus, or discrete math.
- Biochemistry must also not be scheduled on the same day as electrotechnology of fysics.
- Calculus should not be written the same day as electrotechnology or geography.
- Discrete math should also not be written the same day as electrotechnology or geography.
- Fysics and geography must be written on separate days.

a) Illustrate the conditions above visually using a graph. Explain how a graph coloring of the graph corresponds to a valid scheduling.

b) How many days are needed at least to schedule all the exams? Give an example of a schedule using this minimal number of days, and explain why fewer days will not suffice.

Problem 4

a) Compute the greatest common divisor of 198 and 54.
b) What condition does the integer N have to satisfy, in order that the equation $198x + 54y = N$ should have an integer solution?
c) Find the general integer solution to the equation $198x + 54y = 1800$.
d) How many pairs of *positive* integers (x, y) are there, such that the inequalities

$$1790 < 198x + 54y < 1810$$

hold?

Problem 5

Let a_n be the number of n-digit numbers, where every digit is a one, a two or a three, and the digit 1 occurs an even number of times.

a) Compute a_1 and a_2 by listing all such one- and two-digit numbers.
b) Derive a recusrive formula for the number sequence a_n, by finding a connection between the numbers a_n and a_{n-1}.
c) Derive a closed formula for a_n. (Hint: You may want to use the recursive formula that you found in part (b), mathematical induction, and/or geometric sums.)