

RETAKE EXAM, FOUNDATIONS OF DISCRETE MATHEMATICS, MS-A0402

- Time: 6.6.2025, 13:00 - 16:00
- Equipment: One sheet (A4) of hand-written notes, written on one side only.
- Answer each question on a separate page. Each question is worth 4 points.
- Motivate all solutions carefully. Answers without motivation give no points, unless specifically stated.
- Good luck, have fun! /Ragnar

QUESTION 1

Determine using truth tables whether the following statements are tautologies or not. [1p/part]

- (a) $(p \rightarrow \neg p) \leftrightarrow (q \rightarrow \neg q)$
- (b) $(p \rightarrow \neg q) \leftrightarrow (q \rightarrow \neg p)$
- (c) $(p \rightarrow q) \leftrightarrow (\neg p \rightarrow \neg q)$
- (d) $((p \rightarrow q) \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$

QUESTION 2

Consider the permutation

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 1 & 9 & 5 & 8 & 7 & 6 \end{pmatrix}$$

- a) Write it as a product of disjoint cycles. [2p]
- b) Find a positive integer k such that π^k is the identity permutation. [2p]

QUESTION 3

How many distinct ways are there to arrange the letters in the word “COMBINATORICS” such that the vowels (O, I, A, O, I) are always together?¹

QUESTION 4

- (a) Find all integer solutions to the equation $12x + 15y = 999$. [2p]
- (b) How many non-negative integer solutions does the equation have? [2p]

QUESTION 5

For an integer $k \geq 1$, the k -dimensional hypercube graph Q_k is defined as the graph whose vertex set is $\{0, 1\}^k$, and where there is an edge between two binary strings if they differ in exactly one position.

- (a) Draw Q_2 and Q_3 . [1p]
- (b) Determine the number of edges in Q_k as a function of k . [1p]
- (c) A *Hamiltonian cycle* in a graph $G = (V, E)$ is a cycle that visits every node in the graph exactly once. Prove that, for every $k \geq 2$, there is a Hamiltonian cycle in Q_k . [2p]

¹Hint: The word “COMBINATORICS” consists of ten distinct letters, with the letters C, I, and O repeated twice.