Instructions
Exam aids:

- You are allowed to bring **one (1) hand-written A4 paper** (both sides of the paper can be used) with formulas, notes, etc., to be used during the exam.
- You are **not** allowed to use any type of calculator.

Exam structure
The exam consists of two types of problems: multiple-choice problems (Problem 1), and main problems (Problems 2 – 6).

- For the multiple choice problems, only the answer is needed.
- **For the main problems, you have to provide complete solutions with well-motivated steps in all of your calculations, and you have to motivate your answers.**
- The total number of available points are 60 points. For a passing grade, you need to get at least 30 points.

How to hand in solutions

- Start the solution of each new problem at the top of a new sheet. For the multiple-choice problems, you can write all answers on the same sheet.
- **Number all of your solution sheets** and hand them in in sorted order.

Good luck!

Multiple Choice Problems

Problem 1.1 (2.5p)
Let $h[k]$ be the impulse response of a linear, time-invariant, causal, and BIBO stable filter.

Select all the **TRUE** statements:

a) The impulse response is absolutely summable, that is,

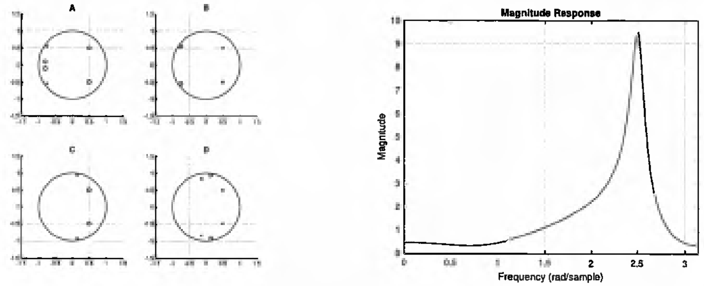
$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty.$$

b) $h[n] = 0$ for all $n < 0$.

c) All zeros of the transfer function are inside the unit circle.

d) All poles of the transfer function are inside the unit circle.

Problem 1.2 (2.5p)
Which pole-zero plot corresponds to the given magnitude response?



a) A

b) B

c) C

d) D

Problem 1.3 (2.5p)
Let the input-output relationships be defined as below. Select all resulting systems that are BIBO stable:

a) $y[n] = \frac{1}{2}y[n-1] + x[n] - \frac{1}{2}x[n-2]$

b) $y[n] = 4y[n-1] + 2x[n] - \frac{1}{2}x[n-2]$

c) $y[n] = \cos[\frac{\pi}{4}] \cdot x[n]$

Problem 1.4 (2.5p)
Let $X(z)$ and $H(z)$ be z-transforms of $x[n]$ and $h[n]$, respectively. Let $a \in \mathbb{R}$ and define the delayed sequence $h_2[n] = h[n-k]$ for some k. Then, the z-transform of $ax[n] * h_2[n]$ can be written as:

a) $aX(z)H(z^{-k})$

b) $az^{-k}X(z)H(z)$

c) $X(az)H(z^{-k})$

d) $X(az) + H(z-k)$

Main Problems

Problem 2 (10p)
Consider the system defined by the difference equation

$$y[n] = -0.8y[n-1] + x[n] + 0.3x[n-1].$$

a) (2.5p) Draw the system on Direct Form I.

b) (2.5p) Compute the transfer function $H(z)$.

c) (5p) Given the input $x[n] = \delta[n]$, what is the output $y[n]$?

Problem 3 (10p)
Consider the following z-domain response:

$$H(z) = \frac{\frac{1}{2} - \frac{1}{2}z^{-1}}{\frac{4}{3} + z^{-1}}.$$

a) (2.5p) Compute poles and zeros and plot the pole-zero plot.

b) (5p) Sketch the approximate magnitude response based on the pole zero plot.

c) (2.5p) Is the filter FIR or IIR? What is the filter type (e.g., lowpass)? Is it stable? Is it minimum or maximum phase? Is it invertible? Provide a reasoning for each attribute.

Problem 4 (10p)
A new measurement device in your laboratory is known to have an unwanted effect that can be modeled as the linear filter

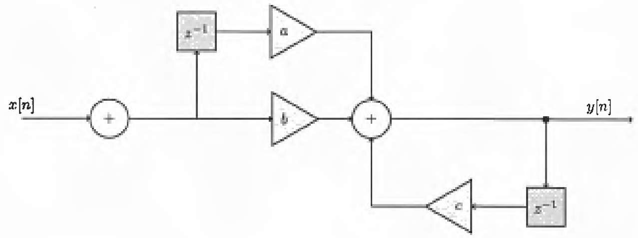
$$y[n] = ax[n] + bx[n-1],$$

where a and b are unknown real coefficients. You test the sensor by giving it a sequence $\{1, 2, 3\}$ for which the (filtered) response is $\{1, 0, -1, -6\}$.

a) (5p) Find the coefficients a and b.

b) (5p) Is it possible to design a stable and causal filter which would reverse the effects caused by the measurement device? Motivate your answer.

Problem 5 (10p)
Consider the following LTI block diagram:



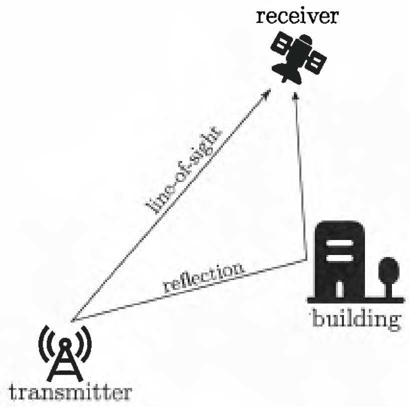
Let $a = 0.5$, $b = 2$, and $c = 1/3$.

a) (2.5p) Compute the z-domain response $H(z)$.

b) (5p) Draw the system in Direct Form II.

c) (2.5p) Is the system stable? Is the system invertible?

Problem 6 (10p)
Consider a wireless link illustrated in the figure below:



The transmitted signal arrives in two paths: a direct line-of-sight (LOS) path and slight reflection from a building nearby. The LOS path arrives at the receiver after N samples from the moment the signal gets transmitted. The reflected path travels a longer distance and arrives at the receiver 6 samples later. Additionally, the reflection absorbs 50% of the transmitted energy.

a) (5p) Find the difference equation describing the wireless link, and derive the corresponding frequency response $H(e^{j\omega})$.

b) (5p) The wireless link can be used for transmitting a signal using any (normalized) frequency ω in the interval $(0, \pi/2]$. What frequency in this interval is the best to transmit at as to maximize received energy? What frequency is the worst? Motivate your answers.