MATRIX ALGEBRA
MS-A0011
BRZUSKA/MILLER
COURSE EXAM/EXAM, 2.12.2025

This exam is both, a *course exam* (if you have been registered for the course) and a plain exam (if you are not registered for the course, but only for the exam). The use of calculators or other notes is not permitted. Each question is worth 6 points, the exam has 6 questions, so the maximal number of points is 36.

Problem 1
Find the permutation matrix P, such that $PA = LU$ holds, where

$$A = \begin{pmatrix} 4 & 9 & 2 \\ 2 & 3 & 2 \\ 7 & 2 & 4 \end{pmatrix}, \quad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{7}{2} & -\frac{17}{6} & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 2 & 3 & 2 \\ 0 & 3 & -2 \\ 0 & 0 & -\frac{26}{3} \end{pmatrix}.$$

Problem 2
Determine all solutions to the equation

$$Ax = b,$$

where

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 5 \\ 1 & 0 & 3 \end{pmatrix}, \quad b = \begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix}.$$

Problem 3
Determine all parameters $\alpha \in \mathbb{R}$ such that the columns of the matrix

$$A = \begin{pmatrix} 1 & 1 & \alpha \\ \alpha & 1 & 1 \\ 1 & \alpha & 1 \end{pmatrix}$$

are linearly independent. Justify your answer!

Problem 4
Let S be the symmetric matrix

$$S = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Diagonalize S, i.e., find an invertible matrix V and its inverse V^{-1} such that $V^{-1}SV$ is diagonal. For the matrices V, S and V^{-1} which you find, check that $V^{-1}SV$ is indeed diagonal.

Problem 5
Let

$$A = \begin{pmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{pmatrix}.$$

Determine $\lim_{k\to\infty} A^k$, i.e., the value of A^k, as k tends to infinity.

Problem 6
Let

$$S = \frac{1}{11}\begin{pmatrix} 10 & 3 & -1 \\ 3 & 2 & 3 \\ -1 & 3 & 10 \end{pmatrix}.$$

a) Show that $S^2 = S$.

b) Is the matrix S a reflection? Justify your answer!

HV