T-61.5100 Digital Image Processing

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1. Explain briefly, with 20–40 words or a mathematical definition, the following concepts or abbreviations: 6p.

(i) homomorphic filtering
(ii) ringing
(iii) transform coding in image compression
(iv) Hough transform
(v) Haar's scaling and wavelet functions
(vi) duality of morphological operations

2. We'll be studying the "classical" discrete Laplace operators shown in figures (a) and (b) and some alternative forms shown in figures (c) and (d). (i) Show that one or some of the masks (a)–(d) can be formed as a discrete approximation of the continuous-valued Laplace operator

$$\Delta = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

(ii) Show that one or some of the masks can be interpreted as the difference between a low-pass filtered and the original image. (iii) Show that one or some of the masks can be implemented as two consecutive one-dimensional filterings. Based on that, calculate the frequency response(s) $H(u, v)$ in the case of an $N \times N$-sized image. Sketch the shape of $H(u, v)$ along lines (1): $u \in [0, N/2], v = 0$ and (2): $u = v \in [0, N/2]$. (iv) Show that one or some of the masks can be implemented as the difference between the result of two consecutive one-dimensional filterings and the original image. Calculate and sketch the frequency response(s). (v) Show that the mask (d) can be formed from two of the other masks (a)–(c) and calculate and sketch its frequency response. (vi) Calculate the responses of all the masks for the image patterns (e)–(g). Discuss the appropriateness of all masks (a)–(d) for the discrete Laplace operator. 6p.

0	1	0
1	−4	1
0	1	0

(a)

1	1	1
1	−8	1
1	1	1

(b)

−1	2	−1
2	−4	2
−1	2	−1

(c)

2	−1	2
−1	−4	−1
2	−1	2

(d)

1	1	1
1	1	1
1	1	1

(e)

−1	1	−1
−1	1	−1
−1	1	−1

(f)

1	−1	1
−1	1	−1
1	−1	1

(g)

Turn page!

3. Below there is one row of an image of 8 gray levels and width of 15 pixels. (i) Form a lossless run-length coding for the bit planes of the binary code. It is assumed that each row starts with a 0-valued run and that all run lengths are coded with four bits. (ii) Similarly, form lossless run-length coding based on the bit planes of Gray code. (iii) Calculate the average number of bits needed per pixel for both codings. Also calculate the compression ratios relative to the original representation. (iv) Calculate the relative redundancy of the original representation relative to the better run-length coding. (v) Evaluate the results. How could this coding be further enhanced? (vi) That type of redundancy is here being removed and what other types of redundancy do exist? 6p.

0 0 0 1 1 2 2 5 4 4 7 7 6 6 6

4. (i) Sketch the chromaticity diagram by using as its extreme points the following (x, y, λ) values: $(0.75, 0.25, 780\,\text{nm})$, $(0.05, 0.8, 520\,\text{nm})$, $(0, 0.65, 505\,\text{nm})$ and $(0.2, 0, 380\,\text{nm})$. Explain how the hue, saturation and intensity axes are located in the diagram. (ii) Let us suppose that a radiating body produces the (unitless) tristimulus values $(X, Y, Z) = (110, 70, 20)$. Calculate the normalized tristimulus values or trichromatic coefficients (x, y, z) and place the calculated (x, y) in the above chromaticity diagram. (iii) Place the names of the spectrum's colors and white in their correct places in the chromaticity diagram. Also name the color of the previous question. (iv) Consider any two valid colors c_1 and c_2 with coordinates (x_1, y_1) and (x_2, y_2) in the chromaticity diagram. Derive the necessary general expressions for computing the relative percentages of colors c_1 and c_2 composing a given color c that is known to lie on the straight line joining these two colors. (v) A third color c_3 with coordinates (x_3, y_3) is then added. Explain and visualize how the relative percentages would now be calculated. (vi) Explain why no combination of three radiating bodies can be used to produce all the colors producible from the spectrum's colors. 6p.