Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.3001 Logic in Computer Science: Foundations

Examination, January 3, 2007

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 $(10p)$

(a) Define the following concepts: *truth assignment*, *sound proof system*, and *ground term*. $(3 \times 2p)$

(b) What is meant by the notation $\phi \equiv \psi$?

Prove in detail that $\phi \equiv \psi$ if and only if $\models \phi \leftrightarrow \psi$. $(4p)$

Assignment 2 $(10p)$ Prove the following claims using semantic tableaux:

(a) $\models (A \land B) \lor (\neg A \land C) \rightarrow (A \rightarrow B) \land (\neg A \rightarrow C)$

(b) $\{\forall x(P(x) \rightarrow R(x)), \forall x(\neg Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 $(10p)$ Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\forall x A(x) \land \forall x B(x) \rightarrow \forall x(A(x) \land B(x))).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 $(10p)$ Let us represent strings "", "*a*", "*b*", "*aa*", "*ab*", "ba", "bb", ... that consist of letters a ja b using ground terms

$$e,\ a(e),\ b(e),\ a(a(e)),\ a(b(e)),\ b(a(e)),\ b(b(e)),\ \ldots,$$

built of a constant symbol e, which represents the empty string "", and unary functions $a(x)$ and $b(x)$, that append the respective letter a or b at the beginning of a string x. Thus $a(b(e))$ is interpreted as $a(b(``")) = a(``b") = ``ab"$.

(a) Define predicate $O(x)$ = "the number of occurences of a in the string x is odd" using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model $S \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models O(a(b(a(e)))).$$

Assignment 5 $(10p)$

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}\,(B)\ \texttt{then}\ \{C_1\}\ \texttt{else}\ \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Diff`:

```
v=0 ; z=0 ; while(!(z==y)) {z=z+1 ; v=v-1} ; v=v+x.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Diff}\ [\texttt{v==x-y}].$$

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.