

Mat-1.1631 Mathematics 3-I

Please fill in the required information into each paper sheet.
Only ordinary calculators are allowed.

1. (a) Is the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ positive definite? (1p)

(b) Find the Cholesky LL^T factorization for this matrix.

(c) Solve the linear system $Ax=b$ using the Cholesky factorization, if $b=[2 \ -2 \ 3]^T$. (3p)

(2p)

2. Are the following statements true (false)? If true, give an explanation (reference to a theorem, a proof), otherwise give a contrary example, a correct statement or a proof.

(a) If the product of matrices A and B is equal to zero, then obligatory $A=0$ or $B=0$ (2p)

(2p)

(b) $(AB)^T=A^T B^T$ (A and B are matrices)

(c) If a square matrix has proportional rows(columns), then its determinant is zero (2p)

3. (a) Evaluate $\left(\frac{1-i}{1+i}\right)^8$ (2p)

(b) Sketch the set $|z-i|=|z-1|$ in the complex plane

(c) Find all zeros of the function $f(z) = \sin z$ (2p)

4. (a) Is the function $f(z) = \bar{z}$ differentiable? (3p)

(b) Find all points where the mapping $w=(z^2-i)^2$ is not conformal (3p)

5. Evaluate (showing the details) the following integrals

(a) $\int_C \cos z dz$, C : the semicircle $|z|=\pi$, $x \geq 0$, from $-\pi i$ to πi (3p)

(b) $\oint_C \frac{dz}{z^2+1}$, C : $|z+i|=1$, counterclockwise (3p)