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T-61.3040 Statistical Modeling of Signals

Final Exam 12.1.2007

You may use a non-programmable calculator and basic mathematical tables (no tables containing material directly associated with the course)

1. (max 6p) Explain *briefly* the following topics without unnecessary detail.
i) Autocorrelation. (2p)
ii) Power spectrum. (2p)
iii) Signal subspace. (2p)

2. (max 6p) When you dial a telephone number on a tone-dial phone, you will hear a tone corresponding to each digit you dial. The tone contains two sinusoids at different frequencies. The sinusoids are picked from two groups of four sinusoids so that there is exactly one sinusoid from each group. The frequencies of the sinusoids are known constants. In this manner, 16 different tones can be constructed. The amplitudes of the sinusoids can be assumed to be equal. Suppose you have to design a digital device receiving digitized audio signals. The device should be able to detect which keys have been pressed, using the tones contained in the audio signal. You known that the signal is sampled at 8kHz rate and in any tone the frequencies of the sinusoids are at least 268 Hz apart.

To begin, you estimate the frequencies using a periodogram. Estimate roughly how long you must press a key so that the periodogram can resolve the frequencies. (2p)

Design a better method than the periodogram. Your method should take into account possible noise and everything that is known about the signal. Justify your method. (4p)

3. (max 6p)

Answer the following propositions either "true" or "false": you may also leave any of them unanswered. A correct answer gives 1 point, a wrong answer -1 points, and a missing answer zero points. No need to justify your answers.
a) A real-valued WSS process $x(n)$ can have the autocorrelations $r_x(0) = 2, r_x(\pm 1) = -1.5, r_x(\pm 2) = 0$.
b) The WSS process $x(n)$ can have an autocorrelation matrix R_x of size $M \times M$ which has $M^2/2$ elements with different values.
c) A strict sense stationary process is always also wide sense stationary.
d) It is possible that an $MA(q)$-process is not a WSS process when $q < \infty$.
e) The conditional variance $\mathrm{Var}(x(n+1)|x(n), x(n-1), \dots)$ of a normally distributed $AR(p)$-process is fully determined by the parameter $b(0)$.
f) If you are Wiener filtering a process $x(n) = d(n) + v(n)$ in which $v(n)$ is zero-mean white noise uncorrelated with the desired process $d(n)$, then it is possible to solve the Wiener filter using only the noise autocorrelation $r_v(k)$ and the autocorrelation $r_d(k)$ of the desired process.

4. (max 6p)

Model a wide-sense stationary process $x(n)$ by filtering unit-variance white noise $v(n)$ through a system

$$H(z) = \frac{b(0)}{1 + a(1)z^{-1} + a(2)z^{-2}}$$

Use estimated autocorrelations $\hat{r}_x(0) = 0.9, \hat{r}_x(1) = -0.8$ ja $\hat{r}_x(2) = 0.6$.