Mat-1.1620 Mathematics 2

Examination: 27.03.2007

No calculators or any tables are allowed in the exam.

Problem 1.

What is the domain of $f(x,y) = \dfrac{x-y}{x^2-y^2}$? Does $f(x,y)$ have a limit as $(x,y) \to (1,1)$?
Can the domain of the function f be extended so that the resulting function is continuous at $(1,1)$?

Problem 2.

Evaluate the following limits.

$$a) \lim_{(x,y)\to(0,\pi/2)} \frac{sin(xy)}{1-x}; \qquad b) \lim_{(x,y)\to(0,0)} \frac{x^2y^{16}}{x^2+y^6}.$$

Problem 3.

Show that the function $u(x,t) = t^{-1/2}e^{-x^2/4t}$ satisfies the partial differential equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ (the one dimensional heat equation).

Problem 4.

Assume that the function $z = f(x,y)$, where $x = 2s+3t$ and $y = 3s-2t$, has continuous partial derivatives of all orders. Find

$$a) \frac{\partial^2 z}{\partial s^2}; \quad b) \frac{\partial^2 z}{\partial s \partial t}; \quad c) \frac{\partial^2 z}{\partial t^2}.$$

Problem 5.

Find the Jacobian matrix $D\mathbf{f}(1,3,3)$ for the transformation of $\mathbb{R}^3$ to $\mathbb{R}^3$ given by

$$\mathbf{f}(x,y,z) = (x^2y, r^2z, y^2-z^2)$$

and use the result to find an approximate value for $\mathbf{f}(0.99, 3.02, 2.97)$.

Problem 6.

The temperature at position (x,y) in a region of xy-plane is $T(x,y) = x^2e^{-y}$. In what direction at the point $(2,1)$ does the temperature increase most rapidly and what is the rate of increase of f in that direction?