Vainikko

Mat-1.140 Funktionaalianalyysin perusteet

Tentti 5.4.2003

1. Present the definitions of
   (a) linear operator
   (b) linear continuous operator
   (c) linear bounded operator
   (d) linear compact operator

   How are notions (b), (c) and (d) related? (Explain why). Illustrate these notions for integral operators.

2. Present the definition of the resolvent set and the spectrum of a linear bounded operator acting in a Banach space. Find the spectrum of the Volterra integral operator $T \in \mathcal{L}(C[0,b])$ defined by

$$(Tx)(t) = \int_0^t \mathcal{K}(t,s)\, x(s)\, ds \quad (0 \le t \le b)$$

   where the kernel $\mathcal{K}(t,s)$ is continuous for $0 \le s \le t \le b$.

3. Formulate and prove the (Banach) theorem about the inversion of $I - T$ where $\|T\| < 1$.

4. Prove that the convex hull of a relatively compact set is relatively compact. (Do not forget to present the definitions of the notions used in the formulation.)