University of Technology Ilkka

Mat – 1.128 **Elements of Discrete Mathematics** (3 cp)
Examination 29.11.2003

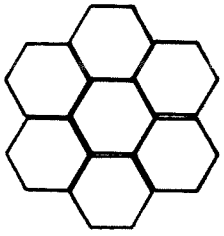
Write first clearly on each sheet of paper
– Mat–1.128 EDM, exam. 29.11.2003
– Student identification number, family name, given names, faculty, signature

1. *a*) Evaluate the sum $\sum_{k=1}^{n} k \binom{n}{k}$.

b) How many non–negative integer solutions has the following equation:

$$x + y + z + t + u + v = 6\,?$$

2. A mosaic window that is designed in the picture is to be made using yellow, red, and blue pieces of glass. We may set the window on the wall at any position in a hole of fitting size and model.



How many different windows is it possible to make? How many, if there are equal numbers of red and blue pieces in the window? Use Polya's Theorem or related results.

3. Find all solutions for the following Diophantine equation

$$3003\,x + 4433\,y = 10010\,,$$

and, specially, the solution with smallest $|\,x\,|$.

4. A Hamming (7, 4)– code has the parity check matrix

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

a) Encode the words 1010, 0111, and 1111.

b) Decode the received words 0101010, and 1001010.

c) We suppose that a symmetric binary channel is used that has an error probability $p = 1\ \%$ for the transmission of one single bit. At which probability will the receiver decode correctly the three–word message sent in part *a*) ?