Teknillinen korkeakoulu

Londen / Mikkola

Mat-1.159 Harmonic Analysis

Mid-term examination 1/2, October 27, 2003 at 12–15 (hall N)

Exercise 1. Show that the Fourier transform is a continuous map from $\mathcal{S}$ to $\mathcal{S}$.

Exercise 2. Show that the Hilbert transform on $\mathcal{S}$ satisfies

$$m(\{x \in \mathbb{R} \mid |Hf(x)| > \lambda\}) \leq \frac{C}{\lambda}\|f\|_{\mathrm{L}^1}$$

for all $f \in \mathcal{S}$ and all $\lambda > 0$.

Exercise 3. Let $\sigma(x,\xi) \in S^0$, and let σ have a compact support in x. Show that then $T_\sigma : \mathrm{L}^2 \to \mathrm{L}^2$ continuously.

Exercise 4. Let Ω be an integrable function on S^{n-1} with zero average. Show that the Fourier transform of P.V.$\frac{\Omega(x')}{|x|^n}$ is given by

$$m(\xi) = \int_{S^{n-1}} \Omega(u) \left[\log\left(\frac{1}{|u\cdot\xi'|}\right) - i\frac{\pi}{2}\operatorname{sgn}(u\cdot\xi')\right] d\sigma(u),$$

where $\xi' := \xi/|\xi|$.