Kevät 2003 Vainikko / Hyvönen

Mat-1.144 Epälineaarinen funktionaalianalyysi

Tentti 13.5.2003

1. Find the first and second derivatives of the Tikhonov functional $f_\alpha(x) = \|Ax - y\|^2 + \alpha \| x - x_0 \|^2$ where $A \in \mathcal{L}(X, Y)$, $x_0 \in X$, $y \in Y$, X and Y are (real) Hilbert spaces. How these derivatives are connected with the minimization problem $f_\alpha(x) \mapsto \min$, $x \in X$, and with the (ill-posed, in general) problem $Ax = y$?

2. Consider the boundary value problem

$$(Au)(x) := \sum_{i=1}^{d} \frac{\partial}{\partial x_i} a_i\left(x, \frac{\partial u(x)}{\partial x_i}\right) + a_0(x, u(x)) = f(x), \quad x \in \Omega,$$

$$u(x) = 0, \quad x \in \partial\Omega,$$

where $\Omega \subset \mathbb{R}^d$ is an open bounded domain. Formulate the conditions under which the operator $A : W_0^{1,p}(\Omega) \to (W_0^{1,p}(\Omega))'$ satisfies the conditions of Browder – Minty theorem.