LONDEN / HYTÖNEN.

COMPLEX ANALYSIS MAT-1.151.
EXAM MAY 28, 2003.

1. SUPPOSE f IS A CONTINUOUS COMPLEX FUNCTION IN AN OPEN SET Ω SUCH THAT

$$\int_{\partial\Delta} f(z)\,dz = 0$$

FOR EVERY CLOSED TRIANGLE $\Delta \subset \Omega$.
SHOW THAT THEN f IS HOLOMORPHIC IN Ω.

2. LET m BE A POSITIVE INTEGER AND

$$P(z) = z^m + a_{m-1} z^{m-1} + \cdots + a_1 z + a_0 ,$$

WHERE $a_0, a_1, \ldots, a_{m-1}$ ARE COMPLEX NUMBERS.
SHOW THAT P HAS PRECISELY m ZEROS IN THE PLANE.

3. LET $U \subseteq \mathbb{C}$ BE AN OPEN SET. LET $f : U \to \mathbb{R}$ BE SUBHARMONIC. LET $\phi : \mathbb{R} \to \mathbb{R}$ BE NONDECREASING AND CONVEX. SHOW THAT $\phi(f(z))$ IS SUBHARMONIC ON U.

4. DETERMINE THE VALUES OF $p \in \mathbb{R}$ FOR WHICH THE INFINITE PRODUCT

$$\prod_{n=1}^{\infty} \left(1 + \frac{1}{n^p}\right)$$ CONVERGES.

5. LET $\Omega \subseteq \mathbb{C}$ BE AN OPEN CONNECTED SET.
LET

$$A^2(\Omega) \overset{\text{def}}{=} \left\{ f \text{ HOLOMORPHIC ON } \Omega \;\middle|\; \int_\Omega |f(z)|^2\,dx\,dy < \infty \right\}.$$

SHOW THAT THERE EXISTS A FUNCTION $K : \Omega \times \Omega \to \mathbb{C}$ SUCH THAT FOR ALL $f \in A^2(\Omega)$ AND ANY $z_0 \in \Omega$,

$$f(z_0) = \int_\Omega K(z_0, z)\, f(z)\,dx\,dy \qquad (z = x + iy)$$