Tfy-44.130 KVANTTIMEKANIIKKA II Whole course exam 07.10.2003

1. Explain (in few lines; use of equations encouraged...) the meaning of the following concepts:

   a) Schrödinger and Heisenberg pictures;
   transformation between these and the equations of motion

   b) Born approximation

   d) Fock space

2. A slowly moving particle scatters from a spherically symmetric potential so that $\delta_l = 0$ to a good approximation for $l \geq 2$.

   (a) Show that the differential cross section is of the form
   $$\sigma(\theta) = A + B\cos\theta + C\cos^2\theta,$$
   and express the coefficients A, B and C in terms of the phase shifts.
   (Hint: Recall that $f_k(\theta) = \frac{1}{k}\sum_l (2l+1)\sin\delta_l e^{i\delta_l} P_l(\cos\theta)$.)

   (b) Calculate the total cross section as a function of A, B and C.

   (c) Let us assume that we have measured the differential cross section in three angles: $\sigma(\pi/2) = \alpha^2$, $\sigma(\pi) = \beta^2$ and $\sigma(0) = \gamma^2$. Calculate the imaginary part of the forward scattering amplitude $\mathrm{Im} f(0)$ as a function of α, β and γ.

3. Derive *Fermi's golden rule* starting from the approximative resonant absorption transition probability
$$\mathcal{P}_{if}(t) = \frac{1}{\hbar^2}|\langle f|A|i\rangle|^2 \left[\frac{\sin\left[\frac{1}{2}(\omega - \omega_{fi})t\right]}{\frac{1}{2}(\omega - \omega_{fi})}\right]^2, \qquad \text{for } \omega \approx \omega_{fi}$$
for a long-lasting harmonic perturbation. Motivate the approximations you make, and explain the situation to which Fermi's golden rule applies.

4. (a) For an arbitrary complex number α, find the normalized boson state $|\alpha\rangle \in \mathcal{H}^+$ that is an eigenstate of an annihilation operator with eigenvalue α: $a|\alpha\rangle = \alpha|\alpha\rangle$. Such a state $|\alpha\rangle$ is called a *coherent state*. Is $|\alpha\rangle$ an eigenstate of $\hat{N}$?

   (b) Show that a bosonic creation operator $a^\dagger$ does not have normalized eigenstates.

5. Calculate the pair distribution function
$$g_{\sigma\sigma'}(\mathrm{x} - \mathrm{x}') = \left(\frac{2}{n}\right)^2 \langle\psi_\sigma^\dagger(\mathrm{x})\psi_{\sigma'}^\dagger(\mathrm{x}')\psi_{\sigma'}(\mathrm{x}')\psi_\sigma(\mathrm{x})\rangle,$$
where $n = \frac{N}{V}$ is the average particle density, for a homogeneous spin-$\frac{1}{2}$ ideal gas in its ground state. Interpret the result!