Mat-1.187 Special course in dynamical systems

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Write clearly in your exam paper: name of the course, your name, your student number and department. Sign the paper.

Problem 1. Define ergodicity and state the pointwise ergodic theorem. Explain in full detail all the concepts and notations used in the statements.

Problem 2. Let (X_A, σ) be the one-dimensional golden mean subshift. Define the topological entropy in this context and prove that $h_{top} = \log\left(\frac{1+\sqrt{5}}{2}\right)$ *from first principles* i.e. without using tools like the Perron-Frobenius theory or the Spectral Radius Theorem.

Problem 3. Write down the rule table and Cayley table for the elementary cellular automaton Rule 30 (rule on binary triples). Analyze the left/right permutivity/subpermutivity by finding invariant subalphabets. Does it preserve the uniform Bernoulli measure? Why/why not?

Problem 4. Let $\left(X^{(W,P)}, \sigma_h, \sigma_v\right)$ be the dimer system i.e. a SOFT on $\{0,1,2,3\}^{\mathbf{Z}^2}$ defined by the allowed patterns

$$\begin{array}{cc} -3 & \\ 0 & 2 \end{array} \qquad \begin{array}{cc} 3 & \\ 1 & -2 \end{array} \qquad \begin{array}{cc} -3 & \\ 2 & -2 \end{array} \qquad \begin{array}{cc} -3 & \\ 3 & -2 \end{array}$$

where $-s$ denotes any symbol but s. Find subsets of dimer configurations providing increasing lower bounds for the topological entropy. Compute also a non-trivial upper bound for the topological entropy i.e. better than the one provided by the two-dimensional full shift.
Let $P_{k,l}$ be the set of dimer configurations that are horizontally of period k and vertically of period l. Show that its cardinality grows at the exponential rate h_{top}.