Final exam 19.11.2003

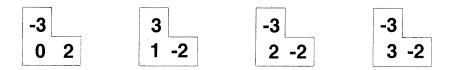
Write clearly in your exam paper: name of the course, your name, your student number and department. Sign the paper.

**Problem 1.** Define ergodicity and state the pointwise ergodic theorem. Explain in full detail all the concepts and notations used in the statements.

**Problem 2.** Let  $(X_A, \sigma)$  be the one-dimensional golden mean subshift. Define the topological entropy in this context and prove that  $h_{top} = \log\left(\frac{1+\sqrt{5}}{2}\right)$  from first principles i.e. without using tools like the Perron-Frobenius theory or the Spectral Radius Theorem.

**Problem 3.** Write down the rule table and Cayley table for the elementary cellular automaton Rule 30 (rule on binary triples). Analyze the left/right permutivity/subpermutivity by finding invariant subalphabets. Does it preserve the uniform Bernoulli measure? Why/why not?

**Problem 4.** Let  $(X^{(W,P)}, \sigma_h, \sigma_v)$  be the dimer system i.e. a SOFT on  $\{0, 1, 2, 3\}^{\mathbb{Z}^2}$  defined by the allowed patterns



where -s denotes any symbol but s. Find subsets of dimer configurations providing increasing lower bounds for the topological entropy. Compute also a non-trivial upper bound for the topological entropy i.e. better than the one provided by the two-dimensional full shift.

Let  $P_{k,l}$  be the set of dimer configurations that are horizontally of period k and vertically of period l. Show that its cardinality grows at the exponential rate  $h_{top}$ .