Kyl-0.108 Nanophysics exam fall 2003
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Note that the exam paper has two sides. Good luck!

1. a) Derive the conditions for Coulomb blockade in a single electron transistor. Sketch the blockaded region on the plane spanned by the gate and drain-source voltages.
b) Assume a small (infinitesimal) current bias on a SET. When is it possible to use the SET as a voltage amplifier (gain > 1)? Take the voltage over the SET as the output voltage.

2. Calculate the eigenenergies and the eigenstates of the two-state quantum system, whose Hamiltonian can be written in spin-1/2 notation as

$$H = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x,$$

where B_x and B_z are the x and z components of the 'magnetic field', and σ_x and σ_z are Pauli spin matrices. Write down B_x and B_z for a Josephson charge qubit (Cooper pair box) near the degeneracy point of two charge states.

3. (a) Derive the current-voltage (IV) characteristics of a tunnel junction between two normal metals with constant density of states (DOS). Show that this ohmic dependence is temperature independent. Use the simple energy diagram as the starting point and assume that tunnelling transmission probability is also energy independent in this narrow energy interval. (b) Calculate the IV characteristics with the same assumption about transmission for a tunnel junction between a normal metal (again constant DOS) and a superconductor with ideal BCS DOS, but now at zero temperature. BCS DOS, divided by the corresponding normal state DOS, is given by

$$n(E) = \frac{|E|}{\sqrt{E^2 - \Delta^2}}\theta(|E| - \Delta),$$

where Δ is the energy gap of the superconductor and $\theta(x)$ is the Heaviside step function, unity for $x > 0$ and vanishing for $x < 0$. In both (a) and (b) assume that charging energies play no role. Sketch the IV curves of (a) and (b).

4. The effect of superconductivity (for $E < \Delta$) on the electron energy distribution functions at an ideal normal-superconducting (N-S) contact can be written through two boundary conditions (for quasi-one-dimensional wire, evaluated at the position of the N-S boundary):

$$\partial_x(f(\mu_S - E) - f(\mu_S + E)) = 0$$
$$1 - f(\mu_S + E) - f(\mu_S - E) = 0.$$

Here μ_S is the chemical potential of the superconductor. Make use of this boundary condition and
a) Solve the stationary diffusive-limit distribution function in a quasi-one-dimensional normal-metal wire between a normal-metal reservoir (at $x = 0$, potential $\mu_N = eV \ll \Delta$, temperature $T = 0$) and a superconducting reservoir (at $x = L$, $\mu_S = 0$, $T = 0$),
b) Evaluate the average current in this system (prefactors are not essential).
Hint: define two independent functions $f_s(x, E) = 1 - f(x, E) - f(x, -E)$ and $f_a(x, E) = f(x, -E) - f(x, E)$ and solve for them separately. Remember that at $T = 0$, the Fermi function is a step function, $f^0(E; V, T = 0) = 1 - \theta(E - V)$.

5. Consider a single-channel quantum wire containing two scatterers in series, with transmission probabilities T_1 and T_2. Find the conductance, shot noise and Fano factor for this wire.