# AS-74. 3123 Model-Based Control Systems
# Exam 9. 1. 2006

The questions are available only in English. You can answer in Finnish, Swedish or English. The final grade is given when both the examination and the homework problem have been accepted.

5 problems.

**1.** Consider the control configuration shown in Fig. 1, where the dimensions of the transfer function matrices and signals are appropriate for the general MIMO case.

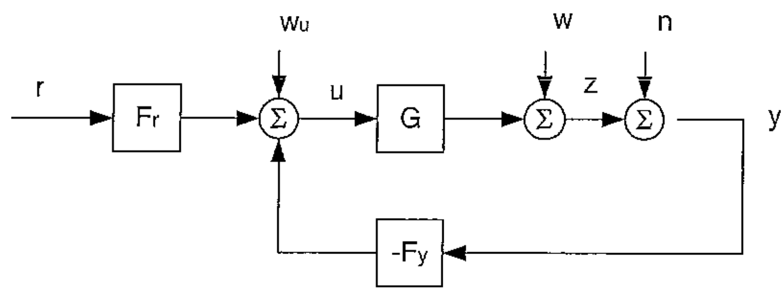


Fig. 1.

**a.** Derive expressions for the signals $z$ and $e = r\text{-}z$ as functions of the input signals $r$, $w$, $n$ and $w_u$ (in Laplace-domain).

**b.** Define the *sensitivity function*, *complementary sensitivity function*, and *closed-loop transfer function*. Prove that $S(i\omega)+T(i\omega)=I$, (the symbols are standard used during the course).

**c.** Consider the SISO case. Analyse the function $S$ and determine where in the complex plane $|S|<1$, $|S|=1$, and $|S|>1$.

**2.** Consider a MIMO system with the transfer function matrix

$$G(s)=\begin{bmatrix} \frac{2}{s+1} & \frac{3}{s+2} \\ \frac{1}{s+1} & \frac{1}{s+1} \end{bmatrix}$$

**a.** What is meant by the *poles* and *zeros* of the system?

**b.** Determine the poles and zeros of the above system.

**c.** What conclusions can be made with respect to control?

**d.** What is meant by the *Relative Gain Array* (RGA)?

**e.** Calculate RGA(0) in the above example case.

**f.** What conclusions can be made with respect to control?

**3.** Consider a SISO-process with the transfer function

$$G(s) = \frac{s + \frac{1}{T_1}}{\left(s + \frac{1}{T_2}\right)\left(s + \frac{1}{T_3}\right)}, \quad T_1 < 0, \quad T_2 < 0, \quad T_3 > 0$$

Explain, what kind of fundamental limitations on control performance can be stated for this system? (Present also some calculations; the formulas in the end of the problems can be of help.)

**4.** Explain briefly the following concepts

**a.** singular values
**b.** $H_\infty$-norm
**c.** internal stability
**d.** "Principle of Optimality"
**e.** LQ-control, LQG control
**f.** "Internal model control" (IMC)

**5.** Consider the 1.order process $G(s) = \frac{1}{s-a}$, which has a realization

$$\dot{x}(t) = ax(t) + u(t)$$
$$y(t) = x(t)$$

so that the state is directly measurable . It is desired to find a control that minimizes the criterion

$$J = \int_0^\infty \left(x^2 + Ru^2\right)dt \qquad (R > 0)$$

Determine the solution and determine the closed loop state equation. Is the closed loop system stable, when the process is i. stable, ii. unstable?

**Some formulas, which might be useful:**

$$\int_0^{\infty} \log \; |S(i\omega)| \, d\omega = \pi \sum_{i=1}^{M} \text{Re}\,(\text{p}_\text{i})$$

$$|W_T(p_1)| \le 1 \quad \Rightarrow \quad \omega_0 \ge \frac{p_1}{1 - 1/T_0}$$

$$|W_S(z)| \le 1 \quad \Rightarrow \quad \omega_0 \le \left(1 - 1/S_0\right) z$$

$$\dot{x} = Ax + Bu, \quad t \ge t_0$$

$$J(t_0) = \frac{1}{2} x^T(T) S(T) x(T) + \frac{1}{2} \int_{t_0}^{T} \left( x^T Q x + u^T R u \right) \; dt$$

$$S(T) \ge 0, \quad Q \ge 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1} B^T S + Q, \; t \le T, \; \text{boundary condition } S(T)$$

$$K = R^{-1} B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2} x^T(t_0) S(t_0) x(t_0)$$