

Mat-1.1220 Basic course in mathematics S2

3. midterm 2008

Please fill in clearly *on every sheet* the data on you and the examination. On *Examination code* mark course code, title and text mid-term or final examination. Study programmes are ARK, AUT, BIO, EST, ENE, GMA, INF, KEM, KJO, KTA, KON, MAK, MAR, PUU, RAK, TFY, TIK, TLT, TUO, YHD.

You may use a calculator.

1. **By using the Lagrange multiplier method** find the minimum value of function $f(x, y) = x^2 + y^2$ under condition $x^2y = 1$. (To obtain full points you are required to use the multiplier method.)

2. Surfaces

$$x^2 + y^2 = 4, \quad y - z + 2 = 0, \quad z = 0,$$

enclose a domain in \mathbf{R}^3 .

- a) Sketch/describe the domain.
 - b) Calculate the volume of the domain.
3. Let $\mathbf{F}: \mathbf{R}^3 \rightarrow \mathbf{R}^3$ be the vector field

$$\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + yx\mathbf{k}.$$

- a) If \mathbf{F} curlfree?
 - b) Is \mathbf{F} sourceless?
 - c) Prove that \mathbf{F} is conservative, and find a scalar potential for \mathbf{F} .
 - d) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ when C is a smooth curve from $(0, 0, 0)$ to $(1, 1, 1)$.
4. Let \mathbf{F} be the vector field $\mathbf{F} = 3y\mathbf{i} - xz\mathbf{j} + yz^2\mathbf{k}$ and let S be the part of the paraboloid $z = \frac{1}{2}(x^2 + y^2)$ that is below the plane $z = 2$. Evaluate $\iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ using Stokes theorem, when S is oriented so that the normal points downwards.