

Write your answers either in Finnish, Swedish or English.

1. Consider a gas of two-level atoms interacting with resonant light of frequency  $\nu$  and intensity  $I$ . The number density of the atoms is  $N$ . Derive the steady-state value for the absorption coefficient

$$\alpha = -\sigma(N_2 - N_1),$$

where  $\sigma$  denotes the cross-section for the stimulated transitions [ $B\rho(\nu) = \sigma I / h\nu$ ] and  $N_2$  and  $N_1$  are the population densities of the upper and lower level, respectively. Write your answer in the form

$$\alpha = \frac{\alpha_0}{1 + I / I_s},$$

and identify the constants  $\alpha_0$  and  $I_s$ . Sketch  $\alpha$  as a function of  $I$  and interpret the result.

2. a) What is a half-wave plate? What does it do and what is it used for?  
b) How thick should a half-wave plate made of mica be in an application where laser light of 633 nm is being used? Appropriate refractive indices for mica are 1.599 and 1.594.
3. Explain laser mode structure. How can a laser be made to operate in a
- (i) single longitudinal mode
  - (ii) single transverse mode.
4. a) Why is it easier to achieve population inversion in a 4-level system than in a 3-level system?  
b) An excimer molecule is stable only in the excited state. Is it good or bad as far as laser operation is concerned? Motivate your answer.  
c) Gas lasers are not pumped with flash lamp. Why is that?  
d) Solid state lasers based on transition metal ions typically have a much broader gain bandwidth than those where the dopant ion is a rare earth metal. Why?  
e) The lasing threshold for a homojunction diode laser is much higher than that of a heterojunction laser. Explain.  
f) CW operation of dye laser requires special procedures. Give a physical explanation.

5. The macroscopic Maxwell's equations for linear, isotropic, dielectric and nonmagnetic medium are

$$\nabla \cdot \mathbf{D} = \rho_f \quad \nabla \cdot \mathbf{B} = 0 \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t},$$

where  $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ ,  $\mathbf{B} = \mu_0 \mathbf{H}$ , and  $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$ .

- (a) Derive the wave equation for the electric field with polarization  $\mathbf{P}$  acting as a source term. Show that a plane wave  $\mathbf{E} = \mathbf{E}_0 \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$  is a solution for the wave equation. [Hint:  $\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$ ].
- (b) How is the refractive index  $n$  related to wave propagation in the medium? Show that in the case considered in (a), the refractive index can be expressed as  $n = \sqrt{1 + \chi}$ .