2nd mid-term exam 15.5.2008

Remember that explaining what you are doing and why is an important part of the grading!

Properties of air:

density: $\varrho_{\text{air}} = 1,23\,\text{kg/m}^3$
(dynamic) viscosity: $\mu_{\text{air}} = 1,79 \cdot 10^{-5}\,\text{Ns/m}^2$

Properties of water:

density: $\varrho_{\text{water}} = 1000\,\text{kg/m}^3$
(dynamic) viscosity: $\mu_{\text{water}} = 1,12 \cdot 10^{-3}\,\text{Ns/m}^2$

Gravitational acceleration: $g = 9,81\,\text{m/s}^2$.

Moment of momentum equation:

$$\Sigma\vec{T} = \dot{m}_{\text{out}}\left(\vec{r}\times\vec{V}\right)_{\text{out}} - \dot{m}_{\text{in}}\left(\vec{r}\times\vec{V}\right)_{\text{in}}$$

$$\vec{r}\times\vec{V} = \pm rV_\theta$$

Euler turbomachine equation:

$$P = \dot{m}\,(\pm UV_\theta)_{\text{out}} - \dot{m}\,(\pm UV_\theta)_{\text{in}}$$

Buckingham Π-theorem:

If an equation involving k variables is dimensionally homogeneous, it can be reduced to a relationship among $k - r$ independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variables.

Criteria for the repeating variables:

1. The number of repeating variables is equal to the number of reference dimensions.
2. All the required reference dimensions must be included within the group of repeating variables.
3. Each repeating variable must be dimensionally independent of the others.

Continuity equation and Navier–Stokes equations will be given if they are required in the exam.

Table 6.1 (Summary of basic, plane potential flows) in the end

1st Problem (6 p.)

Figure 1 depicts the rotor of a small water turbine with the blade outer radius of $r_1 = 0{,}2$ m and the inner radius r_2 of exactly half of that. The height of the blades in the axial direction is 0,1 m. When the rotor revolves at the design speed of 200 rpm, the turbine produces 22 kW of power. In this condition, the exit flow from the rotor is just radial with the velocity of 12 m/s, as indicated in the figure. In which angle α_1 does the stator surrounding the rotor deliver the flow into the rotor? How would the torque change if the rotational speed of the rotor decreased while the incoming flow from the stator remains constant?

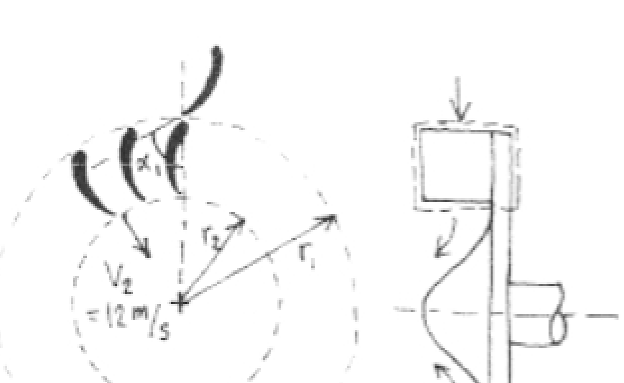

Figure 1: Water turbine of Problem 1

2nd Problem (4 p.)

The equation below describes flow in the x-direction:

$$\varrho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \varrho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

What is this equation and which basic physical principle does it represent? Explain the physical nature of each term in the equation. How can this equation be utilized for applications? Are other equations required to be used in parallel? Is this kind of calculation difficult?

3rd Problem (4 p.)

The equation below is a general form of the flow continuity equation:

$$\frac{\partial \varrho}{\partial t} + \frac{\partial \varrho u}{\partial x} + \frac{\partial \varrho v}{\partial y} + \frac{\partial \varrho w}{\partial z} = 0$$

Reduce this equation to a form sufficient for steady, incompressible and two-dimensional flow and give grounds for the changes.

In addition, if the flow is assumed to be irrotational, the continuity equation just obtained can be converted into a form involving a single scalar function on the basis of mathematics concerning vector fields. Which physical phenomenon is connected to the crucial assumption of irrotationality? How does the relevant scalar function relate to the velocities in the continuity equation? How is the replacement equation actually obtained and what is it like? What is the sense in going into this procedure?

4th Problem (4 p.)

Consider a beam that lies perpendicular to an incoming flow. The cross-section of the beam resembles an ellipse placed longitudinally into the flow (Rankine oval). The flow around the cross-section is symmetrical in two directions and it can be modelled by utilizing three basic potential flows. Which basic flows are needed and what kind of principle is to be used to obtain their combined effect? Write the stream functions of the necessary solution elements. Draw also a draft of the streamlines produced by the combination of the solution elements and indicate the object surface. How can the ratio of the oval length and thickness be controlled to obtain a desired shape?

5th Problem (6 p.)

The possibility of studying the power output of a large, 3-bladed wind turbine experimentally in a wind tunnel is considered. It can be expected that the power P generated depends on the wind speed V, rotor rotational speed ω, rotor diameter D and air density ϱ as well as viscosity μ. Thus,

$$P = F(V, \omega, D, \varrho, \mu)$$

Formulate the corresponding non-dimensional equation and state the similarity requirements between the model test and the actual turbine. What are the upcoming non-dimensional quantities?

How representative results would you expect from the tests if the diameter of the actual turbine is 60 meters, the maximum flow speed in the available tunnel is $V = 25\,\text{m/s}$ and the dimensions of the test section are $3m \times 3m$? With respect to what there might be uncertainties?

Hint: Do not choose angular velocity ω as a repeating variable.

TABLE 6.1
Summary of Basic, Plane Potential Flows

Description of Flow Field	Velocity Potential	Stream Function	Velocity Components[a]
Uniform flow at angle α with the x axis (see Fig. 6.15b)	$\phi = U(x\cos\alpha + y\sin\alpha)$	$\psi = U(y\cos\alpha - x\sin\alpha)$	$u = U\cos\alpha$ $v = U\sin\alpha$
Source or sink (see Fig. 6.16) $m > 0$ source $m < 0$ sink	$\phi = \frac{m}{2\pi}\ln r$	$\psi = \frac{m}{2\pi}\theta$	$v_r = \frac{m}{2\pi r}$ $v_\theta = 0$
Free vortex (see Fig. 6.17) $\Gamma > 0$ counterclockwise motion $\Gamma < 0$ clockwise motion	$\phi = \frac{\Gamma}{2\pi}\theta$	$\psi = -\frac{\Gamma}{2\pi}\ln r$	$v_r = 0$ $v_\theta = \frac{\Gamma}{2\pi r}$
Doublet (see Fig. 6.22)	$\phi = \frac{K\cos\theta}{r}$	$\psi = -\frac{K\sin\theta}{r}$	$v_r = -\frac{K\cos\theta}{r^2}$ $v_\theta = -\frac{K\sin\theta}{r^2}$

[a] Velocity components are related to the velocity potential and stream function through the relationships:

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \qquad v = \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} \qquad v_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad v_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}$$