T-61.5140 Machine Learning: Advanced Probabilistic Methods
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Examination, 29th of August, 2008 from 9 to 12 o'clock.

In order to pass the course and earn 5 ECTS credit points, you must also pass the term project. Results of this examination are valid for one year after the examination date. Information for Finnish speakers: Voit vastata kysymyksiin myös suomeksi, kysymykset on ainoastaan englannin kielellä. Information for Swedish speakers: Du får också svara på svenska, frågorna finns dock endast på engelska.

1. Define the following terms shortly:
a) Bayesian network
b) first-order Markov chain
c) conditional independence
d) Hammersley-Clifford theorem
e) Bayes's theorem
f) likelihood function

2. Given a Bayesian network model $P(X|C_1, C_2, \ldots, C_k)$, derive the corresponding junction tree (state the steps) and calculate the treewidth of the model. Draw the corresponding graphical representation of the original model and the junction tree.

3. Given a Hidden Markov Model for a sequence of observations $Y = (y_1, \ldots, y_t)$, show that the predictive distribution of the observations y_t follows a mixture distribution.

4. A Bayesian network with two discrete variables with two states each is defined with the probability on X_1 as $P(X_1) = (0.4\ 0.6)^T$ and the conditional probabilities $P(X_2 \mid X_1 = 0) = (0.5\ 0.5)^T$, $P(X_2 \mid X_1 = 1) = (1.0\ 0)^T$. Write the (a) Variational Bayes cost function for the variables X_1 and X_2 (ignoring the model parameters) and (b) perform the E-step of the Variational Bayesian inference.

5. Write the probability $p(\mathbf{x})$ for the finite mixture model of exponential distributions, name the parts of the mixture model, and derive the E-step and the M-step of the Expectation-Maximization (EM) algorithm. Hint: The probability for an exponentially distributed random variable can be calculated with the following equation: $p(x \mid \lambda) = \lambda e^{-\lambda x}, x \geq 0$.

6. In a TV game show, the contestant has the chance to win the price if he chooses a right door out of three doors behind which the price is waiting. First, the contestant is asked to select one of the doors. Then, the game show keeper opens another door which does not contain the prize. After that, the contestant is asked if he want to change his selection of the door (out of the two doors that remain unopened). Finally, the contestant wins the prize if it is found behind the selected door. Model the domain with random variables *Price*, *First Selection*, *Game show keeper opens* with a Bayesian network and write the probability table(s) for the network according to the rules of the game. Calculate the probability of winning the prize if the contestant (a) does NOT change his original selection of the door, and (b) changes his original selection.