

S.72-1140 Transmission Methods in Telecommunication Systems

Closed-book exam on Thursday 30.10.2008

1. Probability density function of Rayleigh distributed envelop is given by
 $p(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$, where σ^2 is the variance.
 - a) Derive the cumulative distribution function.
 - b) Find the percentage of time that a signal is 10 dB or more below the RMS value for a Rayleigh fading signal.
2. Consider the signal $x(t) = e^{-\alpha t} u(t)$, $\alpha > 0$
 - a) Is $x(t)$ a power or an energy signal?
 - b) Compute the respective Fourier transform.
 - c) Compute the respective autocorrelation function.
 - d) Show that the autocorrelation function can be written in terms of convolution.
3. Consider matching of a communication channel with $Z_L = 30 + j2\pi f L$, $L = 10 \text{ nH}$ to the source of having impedance $Z_s = 10 + 1/(j2\pi f C)$, $C = 1 \text{ nF}$.
 - a) Describe goodness of matching as a function of frequency.
 - b) Is there any frequency where matching is optimized?
4. An angle modulated signal with the carrier frequency $\omega_c = 2\pi \cdot 10^6$ is described by the equation $x(t) = 10 \cdot \cos[\omega_c t + 0.1 \cdot \sin(\pi 2000t)]$
 - a) Find the average power of the signal when the impedance (resistance) level is 1Ω
 - b) Find the respective frequency deviation Δf
 - c) Find the respective phase deviation $\Delta\varphi$
 - d) Estimate the required transmission bandwidth for $x(t)$
5. For a (6,3) systematic linear block code, the three parity-check digits c_4 , c_5 and c_6 are: $c_4 = d_1 \oplus d_2 \oplus d_3$, $c_5 = d_1 \oplus d_2$ and $c_6 = d_1 \oplus d_3$
 - a) Construct the appropriate generator matrix for this code
 - b) Construct the code(s) generated by this matrix
 - c) Determine the error correction capabilities of the code
 - d) Prepare a suitable decoding table
 - e) Decode the word: 101100

Collection of Formulas

$$C = W_C \cdot \log_2(1 + SNR) \quad \begin{cases} r_{\max} = 2B_T = r_b / n = r_b / \log_2(L) \\ \Rightarrow r_b = 2B_T \log_2(L), L = 2^n \end{cases}$$

$$p_e = Q(\sqrt{\gamma_b}) \text{ (unipolar)}, p_o = Q(\sqrt{2\gamma_b}) \text{ (polar)}$$

$$V(f) = F(v(t)) = \int_{-\infty}^{\infty} v(t) \exp(-j2\pi ft) dt$$

$$G_v(f) = F[R(\tau)] = |V(f)|^2, v(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_0 t), c_n = \frac{1}{T_0} \int_{T_0} v(t) \exp(-j2\pi n f_0 t) dt$$

$$E = \int_{-\infty}^{\infty} |V(f)|^2 df, P_{avg} = \sum_{n=-\infty}^{\infty} |c_n|^2, R(\tau) = \langle x(\tau)x(t+\tau) \rangle$$

$$P_{dB} = 10 \log(P_1/P_2), P_{dB} = 20 \log(V_1/V_2), P_{dBm} = 10 \log(P_1/1mW), \frac{V_g}{V_i} = \frac{Z_g + Z_L}{Z_L}$$

$$\begin{cases} y(t) = Kx(t - t_d) \\ \Rightarrow Y(f) = F[y(t)] = \underbrace{K \exp(-j\omega t_d)}_{H(f)} X(f) \\ G_y(f) = |H(f)|^2 G_x(f) \text{ (= output PDF)} \end{cases}, \begin{cases} l = d_{\min} - 1, t = \lfloor l/2 \rfloor, R_C = k/n \leq 1 \\ d_{\min} \leq n - k + 1 \text{ (repetition codes)} \end{cases}$$

$$\begin{cases} N_R = \int_{-\infty}^{\infty} (\eta/2) |H_R(f)|^2 df \\ = \int_{B_T} (\eta/2) df + \int_{B_T} (\eta/2) df = \eta B_T \end{cases} \quad \begin{cases} P(n, k) = \binom{n}{k} \alpha^k (1-\alpha)^{n-k} \\ \binom{n}{k} = \frac{n!}{k!(n-k)!} \end{cases}$$

$$\begin{cases} B_T = 2|D-1|W, 1 >> D \gg 1 \\ \beta = A_m f_\Delta / f_m \Big|_{A_m=1, f_m=W} = f_\Delta / W \equiv D \\ B_{T, DSB} = 2W, B_{T, SSB} = W \end{cases} \quad \begin{cases} P_e \approx Q\left(\sqrt{\frac{E_b - E_{10}}{\eta} \cos^2 \theta_e}\right) \\ \theta_e = \omega_c \tau \end{cases}$$

$$\begin{cases} x_C(t) = A_C \cos(\omega_C t + \phi(t)) \\ \dot{\phi}_{PM}(t) = \phi_\Delta x(t) \\ \dot{\phi}_{FM}(t) = 2\pi f_\Delta \int_{t_0}^t x(\lambda) d\lambda, t \geq t_0 \end{cases} \quad \phi(t) = \begin{cases} \underbrace{\phi_\Delta A_m}_{\beta} \sin(\omega_m t), PM \\ \underbrace{(A_m f_\Delta / f_m)}_{\beta} \sin(\omega_m t), FM \end{cases} \quad \begin{cases} \gamma = S_R / (\eta W) \\ S_R / N_R = \gamma W / B_T \\ \gamma_b = E_b / N_0 \end{cases}$$

$$y(t) = \begin{cases} v_i(t) & \text{Synchronous detector} \\ A_v(t) - \bar{A}_v & \text{Envelope detector} \\ \phi_v(t) & \text{Phase detector} \\ d\phi_v(t)/dt & \text{Frequency detector} \end{cases}, \quad \begin{cases} x_{AM}(t) = A_C [1 + \mu x_m(t)] \cos(\omega_c t) \\ x_{DSB}(t) = x_m(t) \cos(\omega_c t) \end{cases}$$

$$\begin{cases} \cos^2 \alpha = (1 + \cos 2\alpha)/2 \\ \cos^3 \alpha = (3 \cos \alpha + \cos 3\alpha)/4 \\ (\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2 \\ (\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 \end{cases} \quad \begin{cases} \sin \alpha \sin \beta = 1/2 \cos(\alpha - \beta) - 1/2 \cos(\alpha + \beta) \\ \cos \alpha \cos \beta = 1/2 \cos(\alpha - \beta) + 1/2 \cos(\alpha + \beta) \\ \sin \alpha \cos \beta = 1/2 \sin(\alpha - \beta) + 1/2 \sin(\alpha + \beta) \end{cases}$$

$$Q(k) = \frac{1}{\sqrt{2\pi}} \int_k^\infty \exp\left(-\frac{\lambda^2}{2}\right) d\lambda \quad H(\omega) = V_{out}(\omega)/V_{in}(\omega) = Z_p/Z_i$$

$$\lambda = (m - x)/\sigma \Rightarrow Q(k) = \frac{1}{\sqrt{2\pi}} \int_{\sigma k + m}^\infty \exp\left(-\frac{(x - m)^2}{2\sigma^2}\right) dx$$

$$\begin{cases} P = UI = U^2/R = I^2R \\ R = U/I \end{cases}, \quad \frac{V_g}{V_i} = \frac{Z_g + Z_L}{Z_L}, \quad P_L = V_i I_i \cos \theta$$

$$\cos \theta = R_{tot}/Z_{tot} = R_{tot}/\sqrt{R_{tot}^2 + X_{tot}^2}, \quad X_{tot} = X_g + X_L, \quad R_{tot} = R_L + R_g$$

$$N_{D(PM)} = \int_{-W}^W \frac{\eta}{2S_R} df = \frac{\eta W}{S_R}, \quad N_{D(FM)} = \int_{-W}^W \frac{\eta f^2}{2S_R} df = \frac{\eta W^3}{3S_R}$$

$$S_D/N_D|_{FM} = \frac{f_\Delta^2 S_x}{\eta W^3/(3S_R)} = 3 \left(\frac{f_\Delta}{W} \right)^2 S_x \frac{S_R}{\eta W} = 3 D^2 S_x \gamma \quad S_D/N_D|_{FM,D \gg 1} = \frac{3}{4} \left(\frac{B_T}{W} \right)^2 S_x \gamma$$

$$S_D/N_D|_{PM} = \frac{\phi_\Delta^2 S_x}{\eta W/S_R} = \phi_\Delta^2 S_x \gamma, \text{ where } \phi_\Delta^2 S_x \leq \pi^2 \begin{cases} Q = R\sqrt{C/L} \\ f_0 = (2\pi\sqrt{LC})^{-1} \end{cases}$$

$$\begin{cases} \int \frac{1}{1+x^2} dx = \arctan(x) \\ \int \frac{x^2}{1+x^2} dx = x - \arctan(x) \end{cases} \quad \begin{cases} \prod \left(\frac{t}{\tau} \right) \leftrightarrow \tau \operatorname{sinc} f\tau \\ \Lambda \left(\frac{t}{\tau} \right) \leftrightarrow \tau \operatorname{sinc}^2 f\tau \end{cases}$$

$$\begin{cases} \frac{d^n v(t)}{dt^n} \leftrightarrow (j2\pi f)^n V(f) \\ \int_{-\infty}^t v(\lambda) d\lambda \leftrightarrow \frac{1}{j2\pi f} V(f) + \frac{1}{2} V(0) \delta(f) \end{cases}$$

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{matrix}$$