

## T-79.5202 Combinatorial algorithms

Exam 27 October 2008 / Haanpää (tel. 5243)

Write on each answer sheet the name and code of the course, the exam date, your name, degree programme, student number, your signature and the total number of answer sheets submitted for grading.

### 1. Rank and unrank

Let  $V = \{0, 1, 2, 3, 4, 5\}$  and  $W = V \times V$ . Which 3-subset of  $W$  has rank 379, when the subsets as well as the elements in  $W$  are ordered lexicographically?

### 2. Local search algorithms

Describe simulated annealing and tabu search on a general level. Discuss their similarities and differences. What things have to be decided upon in order to apply these algorithms to solving some problem? Describe a combinatorial problem and explain how simulated annealing and tabu search could be used to solve it.

### 3. Cliques and graph coloring

Given a graph  $G$ , define the *chromatic number*  $\chi(G)$  as the least number of colors needed to color the vertices of  $G$  so that no edge has two endpoints of the same color, and the *clique number*  $\alpha(G)$  as the size of the largest clique in  $G$ .

How do  $\chi(G)$  and  $\alpha(G)$  relate to each other? Give an example graph where  $\chi(G)$  and  $\alpha(G)$  differ by one.

### 4. Burnside's Lemma

Consider a grid of  $2 \times 2$  squares. How many different ways to color the squares with  $c$  colors are there, if two colorings are considered the same if one can be obtained from the other by rotating or mirroring the grid?