S.72-1140 Transmission Methods in Telecommunication Systems

Closed-book Exam on Monday 12.1.2009

1. Consider the AM signal

$$s(t) = A_c[1+\mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$$

produced by a sinusoidal modulating signal of frequency f_m. Assume that the modulation index is $\mu = 2$ (over modulation), and the carrier frequency f_c is much greater that f_m. The AM signal $s(t)$ is applied to an ideal envelope detector producing the output $v(t)$

(a) Sketch $v(t)$

(b) Determine the Fourier series representation of $v(t)$

(b) What is the ration of second-harmonic amplitude to the fundamental amplitude in $v(t)$?

2. Figure 1 shows the circuit diagram of a balanced modulator. The modulating input applied to the device is $x(t)$. The two modulators have the same amplitude sensitivity. Show that the output $s(t)$ of the balanced modulator consists of a double sideband (DSB) modulated signal.

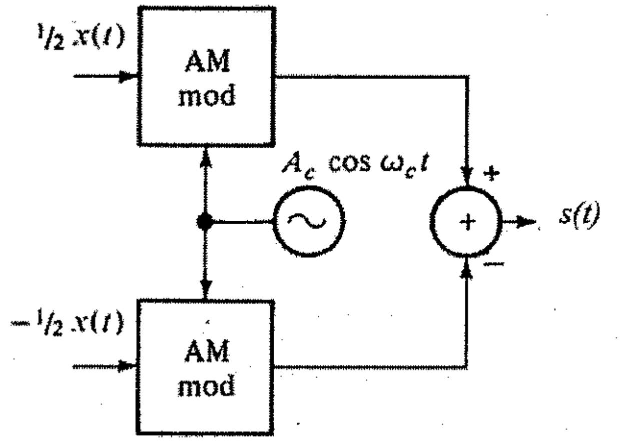

Figure 1

3. An FM signal with modulation index $\beta = 1$ is transmitted through an ideal band-pass filter with midband frequency f_c and bandwidth $5f_m$ where f_c is the carrier frequency and f_m is the frequency of the sinusoidal modulating wave. (a) Determine and sketch the magnitude spectrum of the filter output.

4. If the lowpass components for a bandpass signal are of the form $x_I = 12\cos(6\pi t) + 3\cos(10\pi t)$ and $x_Q(t) = 2\sin(6\pi t) + 3\sin(10\pi t)$

(a) Calculate the Fourier series of the lowpass components $x_I(t)$ and $x_Q(t)$

(b) Calculate the Fourier series of the complex valued signal (complex envelope) $x_Z(t)$

(c) Assuming $f_c = 40\,\text{Hz}$, calculate the Fourier series of $x_c(t)$ where f_c is the carrier frequency and $x_c(t)$ is the composite bandpass signal

(d) Calculate amplitude $x_A(t)$ and phase $x_P(t)$ of the signal

5. (a) Given the signal $x_c(t) = 10\cos(2\pi 10^8 t + 200\cos 2\pi 10^3 t)$, what is its bandwidth in [Hz]?
(b) What is the instantaneous frequency [Hz] of the signal $x_b(t) = 10\cos(20\pi t + \pi t^2)$ at the time instance t = 0?
(c) What is its rate of frequency change [Hz] per second?

Collection of Formulas

$$C = W_C \cdot \log_2(1+SNR)\ ,\ \begin{cases} r_{\max} = 2B_T = r_b/n = r_b/\log_2(L) \\ \Rightarrow r_b = 2B_T\log_2(L), L = 2^n \end{cases},\ r = n\cdot f_S$$

$$C = W_C \cdot \log_2(1+SNR)$$

$$P_{dB} = 10\log(P_1/P_2), P_{dB} = 20\log(V_1/V_2), P_{dBm} = 10\log(P_1/1mW),\ \frac{V_g}{V_i} = \frac{Z_g + Z_L}{Z_i}$$

$$\begin{cases} \text{y(t)=Kx(t - t}_d\text{)} \\ \Rightarrow \text{Y(f)=F [y(t)]=}\underbrace{\text{Kexp(-j}\omega\text{t}_d\text{)}}_{H(f)} X(f) \\ G_y(f) = |H(f)|^2 G_x(f)\ \text{(= output PDF)} \end{cases},\ \begin{cases} l = d_{\min} - 1,\ t = \lfloor l/2 \rfloor, R_C = k/n \le 1 \\ d_{\min}\big|_{\max} = n - k + 1\ \text{(repetition codes)} \end{cases}$$

$$\begin{cases} N_R = \int_{-\infty}^{\infty} (\eta/2)|H_R(f)|^2 df \\ \quad = \int_{B_T} (\eta/2)df + \int_{B_T} (\eta/2)df = \eta B_T \end{cases} \quad \begin{cases} P(n,k) = \binom{n}{k} \alpha^k (1-\alpha)^{n-k} \\ \binom{n}{k} = \frac{n!}{k!(n-k)!} \end{cases}$$

$$c_n = \frac{1}{T}\int_T x(t)\exp(-j2\pi f_0 tn)dt$$

$$\begin{cases} B_T = 2|D-1|W, 1 >> D >> 1 \\ \beta = A_m f_\Delta / f_m \big|_{A_m = 1, f_m = W} = f_\Delta / W \equiv D \\ B_{T,DSB} = 2W, B_{T,SSB} = W \end{cases} \quad \begin{cases} f(x) = a_0/2 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx) \\ a_0 = \pi^{-1}\int_{-\pi}^{\pi} f(x)dx, a_n = \pi^{-1}\int_{-\pi}^{\pi} f(x)\cos(nx)dx \\ b_n = \pi^{-1}\int_{-\pi}^{\pi} f(x)\sin(nx)dx \end{cases}$$

$$\begin{cases} x_C(t) = A_C\cos(\omega_C t + \phi(t)) \\ \phi_{PM}(t) = \phi_\Delta x(t) \\ \phi_{FM}(t) = 2\pi f_\Delta \int_{t_0}^{t} x(\lambda)d\lambda, t \ge t_0 \end{cases} \quad \phi(t) = \begin{cases} \underbrace{\phi_\Delta A_m}_{\beta} \sin(\omega_m t), \text{PM} \\ \underbrace{(A_m f_\Delta / f_m)}_{\beta} \sin(\omega_m t), \text{FM} \end{cases} \quad \begin{cases} \gamma = S_R/(\eta W) \\ S_R/N_R = \gamma W/B_T \\ \gamma_b = E_b/N_0 \end{cases}$$

$$y(t) = \begin{cases} v_i(t) & \text{Synchronous detector} \\ A_v(t) - \overline{A_v} & \text{Envelope detector} \\ \phi_v(t) & \text{Phase detector} \\ d\phi_v(t)/dt & \text{Frequency detector} \end{cases},\ \begin{cases} x_{AM}(t) = A_C[1 + \mu x_m(t)]\cos(\omega_c t) \\ x_{DSB}(t) = x_m(t)\cos(\omega_c t) \end{cases}$$

$$\begin{cases} Q = R\sqrt{C/L} \\ f_0 = (2\pi\sqrt{LC})^{-1} \end{cases}, \quad H(\omega) = V_{out}(\omega)/V_{in}(\omega) = z_p/z_i,$$

$$Q(k) = \frac{1}{\sqrt{2\pi}}\int_k^{\infty} \exp\left(-\frac{\lambda^2}{2}\right)d\lambda \quad \lambda = (m-x)/\sigma \Rightarrow Q(k) = \frac{1}{\sqrt{2\pi}}\int_{\sigma k+m}^{\infty} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)dx$$

$$\begin{cases} P = UI = U^2/R = I^2R \\ R = U/I \end{cases}, \quad \frac{V_g}{V_l} = \frac{Z_g + Z_L}{Z_L}, P_L = V_l I_l \cos\theta$$

$$\cos\theta = R_{tot}/Z_{tot} = R_{tot}/\sqrt{R_{tot}^{\ 2} + X_{tot}^2}, X_{tot} = X_g + X_L, R_{tot} = R_L + R_g$$

$$N_{D(PM)} = \int_{-W}^{W} \frac{\eta}{2S_R} df = \frac{\eta W}{S_R}, \quad N_{D(FM)} = \int_{-W}^{W} \frac{\eta f^2}{2S_R} df = \frac{\eta W^3}{3S_R}$$

$$S_D/N_D\big|_{FM} = \frac{f_\Delta^2 S_x}{\eta W^3/(3S_R)} = 3\underbrace{\left(\frac{f_\Delta}{W}\right)}_{D}^2 S_x \underbrace{\frac{S_R}{\eta W}}_{\gamma} = 3D^2 S_x\gamma, \quad S_D/N_D\big|_{FM,D>>1} = \frac{3}{4}\left(\frac{B_T}{W}\right)^2 S_x\gamma$$

$$S_D/N_D\big|_{PM} = \frac{\phi_\Delta^2 S_x}{\eta W/S_R} = \phi_\Delta^2 S_x\gamma, \text{where } \phi_\Delta^2 S_x \le \pi^2$$

$$\begin{cases} \int \frac{1}{1+x^2}dx = \arctan(x) \\ \int \frac{x^2}{1+x^2}dx = x - \arctan(x) \end{cases} \quad \begin{cases} \Pi\left(\frac{t}{\tau}\right) \leftrightarrow \tau \operatorname{sinc} f\tau \\ \Lambda\left(\frac{t}{\tau}\right) \leftrightarrow \tau \operatorname{sinc}^2 f\tau \end{cases} \quad \begin{cases} \frac{d^n v(t)}{dt^n} \leftrightarrow (j2\pi f)^n V(f) \\ \int_{-\infty}^{t} v(\lambda)d\lambda \leftrightarrow \frac{1}{j2\pi f}V(f) + \frac{1}{2}V(0)\delta(f) \end{cases}$$

$$\begin{cases} \sin\alpha\sin\beta = 1/2\cos(\alpha-\beta) - 1/2\cos(\alpha+\beta) \\ \cos\alpha\cos\beta = 1/2\cos(\alpha-\beta) + 1/2\cos(\alpha+\beta) \\ \sin\alpha\cos\beta = 1/2\sin(\alpha-\beta) + 1/2\sin(\alpha+\beta) \end{cases}, \quad \begin{cases} \cos^2\alpha = (1+\cos 2\alpha)/2 \\ \cos^3\alpha = (3\cos\alpha + \cos 3\alpha)/4 \end{cases} \quad \begin{cases} (\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2 \\ (\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 \end{cases}$$

TABLE A6.5 *Table of Bessel functions*[a]

	$J_n(x)$								
n\x	*0.5*	*1*	*2*	*3*	*4*	*6*	*8*	*10*	*12*
0	0.9385	0.7652	0.2239	−0.2601	−0.3971	0.1506	0.1717	−0.2459	0.0477
1	0.2423	0.4401	0.5767	0.3391	−0.0660	−0.2767	0.2346	0.0435	−0.2234
2	0.0306	0.1149	0.3528	0.4861	0.3641	−0.2429	−0.1130	0.2546	−0.0849
3	0.0026	0.0196	0.1289	0.3091	0.4302	0.1148	−0.2911	0.0584	0.1951
4	0.0002	0.0025	0.0340	0.1320	0.2811	0.3576	−0.1054	−0.2196	0.1825
5	—	0.0002	0.0070	0.0430	0.1321	0.3621	0.1858	−0.2341	−0.0735
6		—	0.0012	0.0114	0.0491	0.2458	0.3376	−0.0145	−0.2437
7			0.0002	0.0025	0.0152	0.1296	0.3206	0.2167	−0.1703
8			—	0.0005	0.0040	0.0565	0.2235	0.3179	0.0451
9				0.0001	0.0009	0.0212	0.1263	0.2919	0.2304
10				—	0.0002	0.0070	0.0608	0.2075	0.3005
11					—	0.0020	0.0256	0.1231	0.2704
12						0.0005	0.0096	0.0634	0.1953
13						0.0001	0.0033	0.0290	0.1201
14						—	0.0010	0.0120	0.0650