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HELSINKI UNIVERSITY OF TECHNOLOGY
Signal Processing Laboratory
S-88.2111 Signal Processing in Telecommunications I (3 cp)

Exam 30 Sept. 2007
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Write in each answer paper your name, department, student number, the course name and code, and the date. Number each paper you submit and denote the total no. of pages. 5 problems, 30 points total. Papers in English only. The BETA mathematical tables can be utilized – you can borrow a copy from the exam supervisor if you do not have your own. A basic calculator can be used (no memory, no graphics).

The homework bonus will be valid for possible future exams too.

1. (1p each) Define and describe *briefly* (2..3 lines of text) the following concepts:

- **a)** Crosstalk
- **b)** Stochastic gradient algorithm
- **c)** Viterbi algorithm
- **d)** OFDM
- **e)** Raised-Cosine (RC) waveform
- **f)** Channel capacity

2. (6p) Matched filters.

Consider a discrete-time receive filter $h_R(k)$ and its frequency response $H_R(e^{j\omega k})$. Assume a simple discrete-time transmit filter:

$$h_T(k) = \delta(k) - 2\delta(k+1) + \delta(k-2) \tag{1}$$

a) (3p) Find the matched-filter receive filter $h_R(k)$ and draw the impulse responses $h_T(k)$ and $h_R(k)$, both the ideal *noncausal* and *causal* versions.

b) (3p) Determine the pulse waveform $g(k)$ at the output of the receive filter either via convolution:

$$g(k) = h_R(k) * h_T(k) = \sum_{l=-\infty}^{\infty} h_R(l) h_T(k-l) \tag{2}$$

or in the frequency domain if you prefer. Plot $g(k)$.

3. Nyquist spectrum (6p):

The EDSL (=Extremely High Speed Digital Subscriber Line) company is facing a major crisis. The circuit design department has failed to produce a transmit filter that meets the Nyquist criterion. The CEO of the company ordered the chief designer (you) to dig up his SPIT1 course and solve the problem for next morning.

Following morning you suggest to use a transmit filter whose frequency response is constant over the frequency range $[-(1-\alpha)W_0, (1-\alpha)W_0]$ and goes linearly to zero in the range $[(1-\alpha)W_0, (1+\alpha)W0]$ (and correspondingly at the negative frequencies).

a) (3p) Draw a sketch of the spectrum magnitude of the transmit filter. Also, convince the CEO that this transmit filter indeed meets the Nyquist criterion.

b) (3p) Solve for the corresponding signal waveform (time-domain impulse response).

(Please turn the page for question numbers four (4) and five (5))

4. (6p) Describe and compare the ZF, the MMSE, and the DFE equalizers. Discuss their advantages and disadvantages for different types of transmission channels, implementation complexities etc.

5. Channel capacity (8 p EXTRA)

In wireline communications, the channel does usually not vary with time but it is frequency dependent. Let us assume that the double-sided channel bandwidth is 2W (baseband channel) and the response is a piecewise constant with frequency but depends on the wireline length L in kilometers, as defined by Eq. (3). The channel noise is AWGN with the double-sided power spectrum $S_n(f) = P_n/(2W)$. $W = 3.8$ kHz and $SNR = P_x/P_n$ is 25 dB.

$$C(f) = \begin{cases} 1, & 0 < |f| < W/2 \\ 1/\sqrt{L}, & W/2 < |f| < 3W/4 \\ 1/(2\sqrt{L},) & 3W/4 < |f| < W \end{cases} \tag{3}$$

a) (4p) Solve for the optimum transmit power spectrum $S_x(f)$ that maximizes the channel capacity when the total transmit power P_x is limited. Draw $S_x(f)$.

b) (4p) Determine the general formula (as a function of L) for this channel's capacity in a simple and compact form. Calculate the numerical values when the channel length is 100m and 200m.

Hints: The optimal power spectrum is obtained with the water-pouring theorem as

$$S_{x,opt}(f) = L - S_n(f)/|C(f)|^2 \tag{4}$$

where the Lagrange multiplier L is determined so that the total transmit power is limited to a constant value. The optimal capacity is then obtained by (double-sided) integration:

$$C = \frac{1}{2}\int_{-\infty}^{\infty} \log_2\left(1 + \frac{S_{x,opt}(f)|C(f)|^2}{S_n(f)}\right) df \tag{5}$$