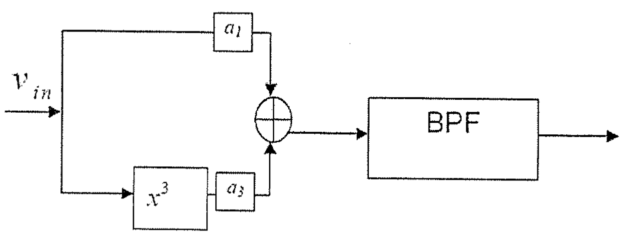
## S.72-1140 Transmission Methods in Telecommunication Systems

*Closed-book Exam on Friday 12.1.2007, 9-12, hall S1*
*You can answer in English, Finnish and/or Swedish.*

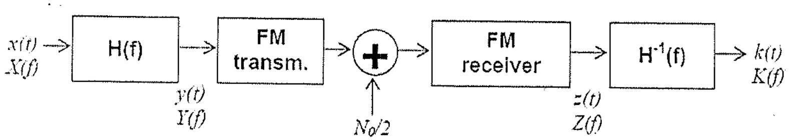
1. Signal $x(t)=\cos(2\pi 200t)$ is sent via FM without pre-emphasis (ilman esikorostusta). Calculate $\left(\frac{S}{N}\right)_D$ when $f_\Delta = 1$ kHz and $S_R = 500\eta$ and the post-detection filter is an ideal BPF passing frequencies in the range of $100\text{Hz} \le f \le 300\text{Hz}$.

2. Figure below presents a DSB modulator that applies a non-linear element having transfer characteristics $u_{out}(t) = a_1 u_{in} + a_3 u_{in}^3$. a) Prove that the device can work as a DSB modulator by determining signal after the BPF. b) What is the condition of $f_c$ in terms of the modulating signal bandwidth W? Assume $v_{in}(t) = x(t) + \cos(\frac{\omega_0}{2}t)$, where x(t) is the modulating signal and $\omega_0$ is the carrier frequency (in radians).




3. Let us suppose that the spectral density of a modulating signal *x(t)* (voice, music) has a double-sided spectrum

$$X(f)=\begin{cases} A_0, |f|<W \\ 0, \text{otherwise} \end{cases}, \text{ W = 15 kHz.}$$




In the ideal radio channel, white Gaussian noise (AWGN) having the spectral density of $N_0/2$ is added to the signal. The pre-emphasis filter $H(f)$ and the de-emphases filter $H^{-1}(f)$ are applied in the system as show in the figure above, and the transfer function of *H(f)* is

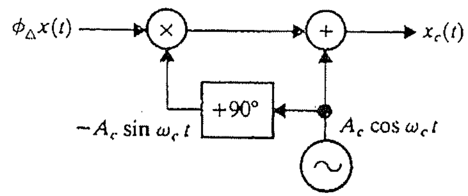
$$H(f)=H_0\left(1+j\frac{f}{f_g}\right), f_g = 5\text{kHz}.$$

a) Calculate the factor $H_0$ so that the input power and the output power are the same in the filter.

b) Calculate the improvement of the SNR (Signal to Noise Ratio) in the output of the receiver when the pre-emphasis and the de-emphasis are used. We assume that the SNR has a large value.

4. Let the total received signal be $v(t)=\cos\omega_c t + \cos[(\omega_c+\omega_i)t]$. a) Sketch the respective phase diagram b) Determine exact expressions for envelope (magnitude) and phase.

5. Consider the narrow band phase modulator shown in the figure below that is used with the modulating signal $x(t)=\text{sinc}^2 2Wt$. Determine the respective spectra for (a) PM. (b) How would you use the device for generating an FM signal? (c) Determine the respective FM spectra.

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**Collection of Formulas**

$$C = W_C \cdot \log_2(1+SNR) \;, \begin{cases} r_{\max} = 2B_T = r_b/n = r_b/\log_2(L) \\ \Rightarrow r_b = 2B_T \log_2(L), L = 2^n \end{cases}, \; r = n \cdot f_s$$

$$C = W_C \cdot \log_2(1+SNR)$$

$$P_{dB} = 10\log(P_1/P_2), P_{dB} = 20\log(V_1/V_2), P_{dBm} = 10\log(P_1/1mW), \; \frac{V_g}{V_l} = \frac{Z_g+Z_L}{Z_L}$$

$$\begin{cases} y(t) = Kx(t-t_d) \\ \Rightarrow Y(f) = F[y(t)] = \underbrace{K\exp(-j\omega t_d)}_{H(f)} X(f) \\ G_y(f) = |H(f)|^2 G_x(f) \;(= \text{output PDF}) \end{cases}, \; \begin{cases} l = d_{\min} - 1, t = \lfloor l/2 \rfloor, R_C = k/n \le 1 \\ d_{\min}\big|_{\max} = n-k+1 \text{ (repetition codes)} \end{cases}$$

$$\begin{cases} N_R = \int_{-\infty}^{\infty} (\eta/2)|H_R(f)|^2 df \\ \quad = \int_{B_T} (\eta/2)df + \int_{B_T} (\eta/2)df = \eta B_T \end{cases} \quad \begin{cases} P(n,k) = \binom{n}{k} \alpha^k (1-\alpha)^{n-k} \\ \binom{n}{k} = \dfrac{n!}{k!(n-k)!} \end{cases} \quad \begin{cases} B_T = 2|D-1|W, 1 >> D >> 1 \\ \beta = A_m f_\Delta / f_m|_{A_m=1, f_m=W} = f_\Delta / W \equiv D \\ B_{T,DSB} = 2W, B_{T,SSB} = W \end{cases}$$

$$\begin{cases} x_c(t) = A_c \cos(\omega_c t + \phi(t)) \\ \phi_{PM}(t) = \phi_\Delta x(t) \\ \phi_{FM}(t) = 2\pi f_\Delta \int_{t_0}^{t} x(\lambda) d\lambda, t \geq t_0 \end{cases} \quad \phi(t) = \begin{cases} \underbrace{\phi_\Delta A_m}_{\beta} \sin(\omega_m t), \text{PM} \\ \underbrace{(A_m f_\Delta / f_m)}_{\beta} \sin(\omega_m t), \text{FM} \end{cases} \quad \begin{cases} \gamma = S_R / (\eta W) \\ S_R / N_R = \gamma W / B_T \\ \gamma_b = E_b / N_0 \end{cases}$$

$$y(t) = \begin{cases} v_i(t) & \text{Synchronous detector} \\ A_v(t) - \overline{A_v} & \text{Envelope detector} \\ \phi_v(t) & \text{Phase detector} \\ d\phi_v(t)/dt & \text{Frequency detector} \end{cases}, \quad \begin{cases} x_{AM}(t) = A_c[1 + \mu x_m(t)] \cos(\omega_c t) \\ x_{DSB}(t) = x_m(t) \cos(\omega_c t) \end{cases}$$

$$N_{D(PM)} = \int_{-W}^{W} \frac{\eta}{2S_R} df = \frac{\eta W}{S_R}, \quad N_{D(FM)} = \int_{-W}^{W} \frac{\eta f^2}{2S_R} df = \frac{\eta W^3}{3S_R}$$

$$S_D / N_D|_{FM} = \frac{f_\Delta^2 S_x}{\eta W^3 / (3S_R)} = 3\underbrace{\left(\frac{f_\Delta}{W}\right)}_{D}^2 S_x \underbrace{\frac{S_R}{\eta W}}_{\gamma} = 3D^2 S_x \gamma, \quad S_D / N_D|_{FM, D>>1} = \frac{3}{4}\left(\frac{B_T}{W}\right)^2 S_x \gamma$$

$$S_D / N_D|_{PM} = \frac{\phi_\Delta^2 S_x}{\eta W / S_R} = \phi_\Delta^2 S_x \gamma, \text{where } \phi_\Delta^2 S_x \leq \pi^2$$

$$\begin{cases} \int \frac{1}{1+x^2} dx = \arctan(x) \\ \int \frac{x^2}{1+x^2} dx = x - \arctan(x) \end{cases} \quad \begin{cases} \prod\left(\frac{t}{\tau}\right) \leftrightarrow \tau \operatorname{sinc} f\tau \\ \Lambda\left(\frac{t}{\tau}\right) \leftrightarrow \tau \operatorname{sinc}^2 f\tau \end{cases} \quad \begin{cases} \frac{d^n v(t)}{dt^n} \leftrightarrow (j2\pi f)^n V(f) \\ \int_{-\infty}^{t} v(\lambda) d\lambda \leftrightarrow \frac{1}{j2\pi f} V(f) + \frac{1}{2} V(0)\delta(f) \end{cases}$$

$$\begin{cases} \sin\alpha \sin\beta = 1/2 \cos(\alpha - \beta) - 1/2 \cos(\alpha + \beta) \\ \cos\alpha \cos\beta = 1/2 \cos(\alpha - \beta) + 1/2 \cos(\alpha + \beta) \\ \sin\alpha \cos\beta = 1/2 \sin(\alpha - \beta) + 1/2 \sin(\alpha + \beta) \end{cases}, \quad \begin{cases} \cos^2\alpha = (1 + \cos 2\alpha)/2 \\ \cos^3\alpha = (3\cos\alpha + \cos 3\alpha)/4 \end{cases} \quad \begin{cases} (\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2 \\ (\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 \end{cases}$$