S.72-1140 Transmission Methods in Telecommunication Systems

Closed-book Exam on Tuesday 19.12.2006, 9-12, halls S3 and S4

1. PAL video signal with bandwidth 0 – 5 MHz is digitized using 8-bit AD-converter. What is the minimum value for sampling frequency and what is the respective bit rate?

2 a. A receiver has bandwidth $W_C = 200\ kHz$ and AWGN level in its input $N_R = 10^{-20}\ \frac{W}{Hz}$. Received signal level is -102 dBm. Calculate SNR in receiver's input and theoretical maximum bit rate.

2 b. Let us suppose that a transmission channel has $BER = 10^{-3}$ and bit errors don't depend on each other. Data is transferred over the channel in packets of 8 bits. Calculate the probability that in a packet

a) all received bits are in error
b) all received bits are correct
c) there are 2 erroneous bits

3. An angle-modulated signal is described by

$$x_c(t) = 10\cos\left[2\pi 10^6 t + 0.1\sin\left(10^3 \pi t\right)\right]$$

a) Consider $x_c(t)$ as a PM signal with $\phi_\Delta = 10$ and find the respective modulating signal $x_1(t)$.
b) Consider $x_c(t)$ as an FM signal with $f_\Delta = 10\pi$ and find the respective modulating signal $x_2(t)$.

4. Let $m_1(t)$ and $m_2(t)$ be two message signals, and let $x_{c1}(t)$ and $x_{c2}(t)$ be the modulated signal corresponding to $m_1(t)$ and $m_2(t)$, respectively.

a) Show that if the modulation is DSB, then $m_1(t) + m_2(t)$ produces modulated signal $x_{c1}(t) + x_{c2}(t)$.
b) Show that if the modulation is PM, then $m_1(t) + m_2(t)$ *does not* produce modulated signal $x_{c1}(t) + x_{c2}(t)$.
c) How would you comment your results?

5 a) A block code consists of the following codes: 10011, 11101, 01110, 00000. How many errors can be detected/corrected by this code? Is this a linear code?

5 b) Signal $x(t) = \cos(2\pi 200 t)$ is sent via FM without pre-emphasis. Calculate $\left(\frac{S}{D}\right)_D$ when $f_\Delta = 1\ kHz$ and $S_R = 500\eta$ and the post-detection filter is an ideal BPF passing frequencies in the range of $100\ Hz \le f \le 300\ Hz$.

Collection of Formulas

$$C = W_C \cdot \log_2(1+SNR)\ ,\ \begin{cases} r_{\max} = 2B_T = r_b/n = r_b/\log_2(L) \\ \Rightarrow r_b = 2B_T\log_2(L), L = 2^n \end{cases}\ ,\ r = n \cdot f_S$$

$$P_{dB} = 10\log(P_1/P_2), P_{dB} = 20\log(V_1/V_2), P_{dBm} = 10\log(P_1/1mW)\ ,\ \frac{V_g}{V_i} = \frac{Z_g+Z_L}{Z_L}$$

$$\begin{cases} \text{y(t)=Kx(t - t}_d\text{)} \\ \Rightarrow \text{Y(f)=F [y(t)]=}\underbrace{\text{Kexp(-j}\omega\text{t}_d\text{)}}_{H(f)}X(f) \\ = H(f)X(f) \end{cases},\ \begin{cases} l = d_{\min}-1,\ t = \lfloor l/2 \rfloor, R_C = k/n \le 1 \\ d_{\min}\big|_{\max} = n-k+1 \text{ (repetition codes)} \end{cases}$$

$$\begin{cases} N_R = \int_{-\infty}^{\infty}(\eta/2)\left|H_R(f)\right|^2 df \\ \quad = \int\limits_{B_T}(\eta/2)df + \int\limits_{B_T}(\eta/2)df = \eta B_T \end{cases}\quad \begin{cases} P(n,k) = \begin{pmatrix} n \\ k \end{pmatrix}\alpha^k(1-\alpha)^{n-k} \\ \begin{pmatrix} n \\ k \end{pmatrix} = \dfrac{n!}{k!(n-k)!} \end{cases}\quad \begin{cases} B_T = 2|D-1|W, 1 >> D >> 1 \\ \beta = A_m f_\Delta / f_m\big|_{A_m=1, f_m=W} = f_\Delta / W \equiv D \\ B_{T,DSB} = 2W, B_{T,SSB} = W \end{cases}$$

$$\begin{cases} x_C(t) = A_C\cos(\omega_C t + \phi(t)) \\ \phi_{PM}(t) = \phi_\Delta x(t) \\ \phi_{FM}(t) = 2\pi f_\Delta \int_{t_0}^{t} x(\lambda)d\lambda, t \ge t_0 \end{cases}\quad \phi(t) = \begin{cases} \underbrace{\phi_\Delta A_m}_{\beta}\sin(\omega_m t), \text{PM} \\ \underbrace{(A_m f_\Delta / f_m)}_{\beta}\sin(\omega_m t), \text{FM} \end{cases}\quad \begin{cases} \gamma = S_R/(\eta W) \\ S_R/N_R = \gamma W/B_T \\ \gamma_b = E_b/N_0 \end{cases}$$

$$y(t) = \begin{cases} v_i(t) & \text{Synchronous detector} \\ A_v(t) - \overline{A_v} & \text{Envelope detector} \\ \phi_v(t) & \text{Phase detector} \\ d\phi_v(t)/dt & \text{Frequency detector} \end{cases}\ ,\ \begin{cases} x_{AM}(t) = A_C[1+\mu x_m(t)]\cos(\omega_c t) \\ x_{DSB}(t) = x_m(t)\cos(\omega_c t) \end{cases}$$

$$N_{D(PM)} = \int_{-W}^{W}\frac{\eta}{2S_R}df = \frac{\eta W}{S_R}\ ,\ N_{D(FM)} = \int_{-W}^{W}\frac{\eta f^2}{2S_R}df = \frac{\eta W^3}{3S_R}$$

$$S_D/N_D\big|_{FM} = \frac{f_\Delta^2 S_x}{\eta W^3/(3S_R)} = 3\underbrace{\left(\frac{f_\Delta}{W}\right)}_{D}^2 S_x \underbrace{\frac{S_R}{\eta W}}_{\gamma} = 3D^2 S_x\gamma\ ,\ S_D/N_D\big|_{FM,D>>1} = \frac{3}{4}\left(\frac{B_T}{W}\right)^2 S_x\gamma$$

$$S_D/N_D\big|_{PM} = \frac{\phi_\Delta^2 S_x}{\eta W/S_R} = \phi_\Delta^2 S_x\gamma, \text{where } \phi_\Delta^2 S_x \le \pi^2$$

$$\begin{cases} \sin\alpha\sin\beta = 1/2\cos(\alpha-\beta) - 1/2\cos(\alpha+\beta) \\ \cos\alpha\cos\beta = 1/2\cos(\alpha-\beta) + 1/2\cos(\alpha+\beta) \\ \sin\alpha\cos\beta = 1/2\sin(\alpha-\beta) + 1/2\sin(\alpha+\beta) \end{cases},\ \begin{cases} \cos^2\alpha = (1+\cos 2\alpha)/2 \\ \cos^3\alpha = (3\cos\alpha + \cos 3\alpha)/4 \end{cases}$$