Helsinki University of Technology
Communications Laboratory

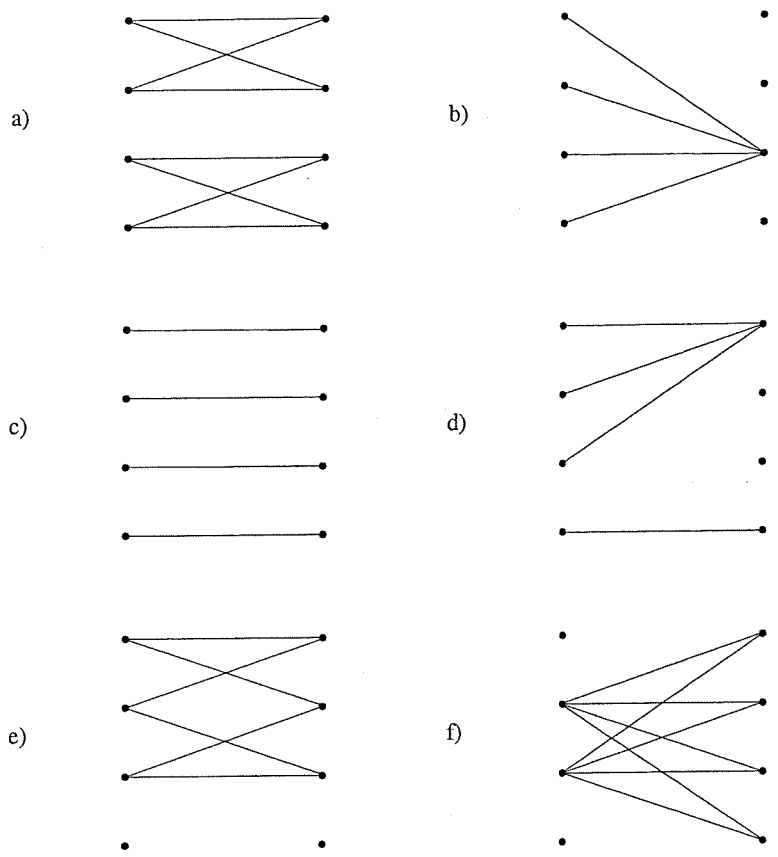
EXAM
14.12.2006

S-72.2410 Information Theory

1. (1p.) Do you intend to give feedback at `http://palaute.ee.hut.fi` no later than on December 18, 2006 (or have you already done so) ? (Yes = 1p. No = 0p.)

2. (8p.) Channel capacity.

   (a) (6p.) What are the capacities of the following six channels? No motivations are needed. In cases a) and e) all edges have probability 0.5, and in case f) all edges have probability 0.25.



   (b) (2p.) If there are three binary symmetric channels available with crossover probabilities $p = 0.40$, $p = 0.50$, and $p = 0.70$, respectively, which one would you choose to use? Motivate.

3. (6p) Entropy.

   (a) (3p.) On a game show, a prize is place behind one of three doors chosen at random with equal probability. There is nothing behind the other two doors. A contestant chooses one of the doors, whereafter the game host opens another door, which does *not* have a prize behind it. (If the contestant chooses the prize door, the host picks one of the two prizeless doors at random.) How much information does the game show host give about the location of the prize?

   (b) (3p.) A box contains two biased coins. Coin 1 has probability of heads p, while coin 2 has probability of heads $1-p$, where $0 < p < 1/2$. An experimenter selects one of the coins at random, that is, with probability $1/2$, and tosses it repeatedly. Let $Z \in \{1, 2\}$ be the outcome of the coin selection, and let $X_1, X_2, X_3, \ldots$ be the outcomes of the coin tosses (heads or tails). Find $I(X_1; X_2|Z)$.

4. (10p.) Data compression.

   (a) (6p.) Consider two discrete memoryless sources. Source 1 has an alphabet of 6 letters with the probabilities 0.3, 0.2, 0.15, 0.15, 0.1, 0.1. Source 2 has an alphabet of 7 letters with the probabilities 0.3, 0.25, 0.15, 0.1, 0.1, 0.05, 0.05. Construct a binary Huffman code and a ternary Huffman code for each source alphabet. Find the average number of code digits per source letter for each code.

   (b) (4p.) Apply the two Lempel-Ziv universal coding methods, LZ77 and LZ78, considered in the course to compress the following strings:

   001100110011001100110011
   010011000111000011110000

   Also the compressed strings should be in binary format.