Helsinki University of Technology
Communications Laboratory

EXAM
4.9.2003

S-72.341 Coding Methods

1. (6p.) Define the following concepts:
   (a) catastrophic (convolutional) code
   (b) 2-error-correcting code
   (c) syndrome vector
   (d) weight distribution
   (e) maximum likelihood decoder
   (f) systematic code

2. (6p.) (Tutorial) Find the length, dimension, and minimum distance for the linear codes defined by the following parity check matrices. The first two codes are binary and the third is over GF(4), where α is a primitive element.

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix},$$

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{H} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 & 0 \end{bmatrix}.$$

3. (6p.) Consider a general weighted code of length n, where each codeword $(x_1, x_2, \ldots, x_n)$ satisfies

$$\left(\sum_{i=1}^{n} i x_i\right) \bmod (n+1) = a.$$

   (a) How many codewords are there in such a code?
   (b) Let $n = 6$ and $a = 1$. Decode the received word $\boldsymbol{y} = 111101$ (with ML decoding assuming a binary symmetric channel).
   (c) Knowing that exactly one error has occurred, in what situations does one have to declare decoder failure? Using this observation, deduce (without formal calculations) for which binary channel this coding is particularly suitable: a) a symmetric channel, or b) an asymmetric channel with either $p(0 \mid 1) \ll p(1 \mid 0)$ or $p(1 \mid 0) \ll p(0 \mid 1)$ ($\ll$ means "much smaller than").

4. (6p.) (Essay) Describe the error control parts of Compact Disc (CD) encoding and decoding. In particular, address the following issues: Why is the encoding process standard, whereas the decoding process is not standardized? Why is an interleaver involved (and what is an interleaver)?