S-72.341 Coding Methods

1. (6p.) (Tutorial) An $(8,4)$ binary linear block code C has parity-check equations

$$v_0 = u_0 + u_1 + u_2,$$
$$v_1 = u_0 + u_1 + u_3,$$
$$v_2 = u_0 + u_2 + u_3,$$
$$v_3 = u_1 + u_2 + u_3,$$

where u_0, u_1, u_2, u_3 are message bits and v_0, v_1, v_2, v_3 are parity-check bits. Codewords are 8-tuples of the form $(v_0, v_1, v_2, v_3, u_0, u_1, u_2, u_3)$.

   (a) Find a generator matrix and a parity-check matrix for this code.

   (b) What is the minimum distance of the code? How many errors can it detect? How many errors can it correct?

2. (6p.) Algebra.

   (a) Express $x^3 + 1 \in \mathrm{GF}(2)[x]$ as a product of binary irreducible polynomials.

   (b) Multiply $x^2 + x + 1$ and $x^2 + x + 2$ in the ring $\mathrm{GF}(3)[x]/(x^3 - 1)$.

3. (6p.) Construct a trellis diagram for the following convolutional encoder, decode the received word $(101, 100, 001, 011, 111, 101, 111, 110)$ using hard-decision Viterbi decoding, and find the original message $\boldsymbol{x}$.

4. (6p.) (Essay) Channels with Feedback. $= ARQ$?