T-61.5130 Machine Learning and Neural Networks
Examination 18th December 2008/Karhunen

You are allowed to have in the examination a collection of mathematical formulas, but not any of the teaching material. (Voit vastata tenttiin myös suomeksi.)

1. Answer briefly (using a few lines) to the following questions or items:

   (a) For what purpose and where is moment term used?

   (b) What is cross-validation?

   (c) Which are the two main criteria for measuring non-Gaussianity?

   (d) Explain briefly ϵ-insensitive cost function.

   (e) Which neural network method is based on competitive learning?

   (f) Explain briefly what is NARX model.

2. Let the error function be

$$\mathcal{E}(\mathbf{w}) = w_1^2 + 10w_2^2,$$

   where w_1 and w_2 are the components of the two-dimensional parameter vector $\mathbf{w}$. Find the minimum value of $\mathcal{E}(\mathbf{w})$ by applying the steepest descent method. Use $\mathbf{w}(0) = [1, 1]^T$ as an initial value for the parameter vector and the following constant values for the learning rate:

   (a) $\alpha = 0.04$

   (b) $\alpha = 0.1$

   (c) $\alpha = 0.2$

   (d) What is the condition for the convergence of this method?

3. Consider solving the XOR problem using an RBF network. In the XOR problem the desired output is 0 for the vectors $(1, 1)^T$ and $(0, 0)^T$ belonging to the first class. For the vectors $(1, 0)^T$ and $(0, 1)^T$ belonging to the second class the desired output is 1. Let us construct a classifier using the basic RBF network, where the radial basis functions are chosen to be multiquadratic type functions

$$\varphi(\| \mathbf{x} - \mathbf{x}_i \|) = [\| \mathbf{x} - \mathbf{x}_i \|^2 + 3]^{1/2}$$

   where $\mathbf{x}_i$ is the i:th training vector. Present how the solution is calculated, and form the equations needed for the solution. Note: you do not need to solve these equations numerically.

4. Consider n zero mean source signals s_i, which have been collected to the vector $\mathbf{s} = (s_1, s_2, \ldots, s_n)^T$. Assume that the joint distribution of the sources is n variables multivariate Gaussian distribution

$$p(\mathbf{s}) = (2\pi)^{-n/2} (\det \mathbf{C})^{-1/2} \exp\left\{-\frac{1}{2}\mathbf{s}^T\mathbf{C}^{-1}\mathbf{s}\right\}$$

CONTINUES ON THE REVERSE SIDE →

Here $\mathbf{C} = \mathrm{E}\{\mathbf{s}\mathbf{s}^T\}$ is the covariance matrix of the vector $\mathbf{s}$ and det means determinant. Assume that the source signals $s_1, s_2, \ldots, s_n$ are mutually uncorrelated. Show that they are mutually statistically independent.

Does this propery hold for other than Gaussian distributed source signals? How about the reverse property: are statistically independent source signals source signals mutually uncorrelated?