S-72.244 MODULATION AND CODING METHODS

Exam 28.8 .2003, 9-12, S4

This is a closed-book exam
– hence no handouts or tutorial material can be present.

1. Bandpass signal can be presented in time domain as

$$v_{bp}(t) = \mathrm{Re}\left\{A(t)\exp\left[j\omega_c t + \phi(t)\right]\right\}.$$

Show that it can be presented in frequency domain as

$$V_{bp}(f) = V_{lp}(f - f_c) + V_{lp}^{*}(-f - f_c).$$

2 a) Let the modulating signal be a square wave that switches periodically between $x(t) = \pm 1$. Sketch $x_c(t)$ when the modulation is AM with $\mu = 0.5$, AM with $\mu = 1$ and DSB. Indicate the envelope with dashed line.

2 b) Suppose a voice signal has $|x(t)|_{\max} = 1$ and $S_x = 1/5$. Calculate the values of S_T and $A_{\max}^2$ needed to get $P_{sb} = 10\,\mathrm{W}$ for DSB and for AM with $\mu = 1$.

3. Suppose an audio signal is modeled as a sum of tones with low-frequency amplitudes $A_m \le 1$ for $f_m \le 1\mathrm{kHz}$ and high-frequency amplitudes $A_m \le 1\mathrm{kHz}/f_m$ for $f_m > 1\mathrm{kHz}$. Use the equations $B = 2M(\beta)f_m, M(\beta) \ge 1$ and $M(\beta) \approx \beta + 2$ to estimate the FM transmission bandwidth required for a single tone at $f_m = 15\mathrm{kHz}$ whose amplitude has been preemphasized at the transmitting end by

$$|H_{pe}(f)| = \left[1 + j\left(\frac{f}{f_{de}}\right)\right] \approx \begin{cases} 1, |f| << \mathrm{B}_{de} \\ \dfrac{jf}{B_{de}}, |f| >> \mathrm{B}_{de} \end{cases}$$

with $\mathrm{B}_{de} = 2\mathrm{kHz}$. Assume $f_\Delta = 75\mathrm{kHz}$ and compare your result with $B_T \approx 210\mathrm{kHz}$.

4. Bandpass gaussian noise with $\sigma_n = 2$ is applied to an envelope detector whose demodulation process is described by $y(t) = A_v(t) - \overline{A}_v$. Find and sketch the probability density function (PDF) of the output *y(t)*, and calculate σ_y.

5. A voice signal having $W = 3$ kHz and $S_x = 1/4$ is to be transmitted via M-ary PCM (uniform quatization). Determine values for M, v and f_s such that $(S/N)_D \ge 40$ dB if $B_T = 16$ kHz.

COLLECTION OF FORMULAS

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

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$$\mathcal{F}[y(t)] = Y(f) = \int_{-\infty}^{\infty} y(t)\exp(-2\pi ft)dt$$

$$\mathcal{F}^{-1}[Y(f)] = y(t) = \int_{-\infty}^{\infty} Y(f)\exp(2\pi ft)df$$

$$\beta_{PM} = \phi_\Delta A_m, \beta_{FM} = A_m f_\Delta / f_m$$

$$\left(\frac{S}{N}\right)_D = \frac{S_x}{1+S_x}\gamma$$

$$\cos(\beta\sin(\omega_m t)) = J_O(\beta) + \sum_{n\,\text{even}}^{\infty} 2J_n(\beta)\cos(n\omega_m t)$$

$$\sin(\beta\sin(\omega_m t)) = \sum_{n\,\text{odd}}^{\infty} 2J_n(\beta)\sin(n\omega_m t)$$

$$P_e = Q\left(\sqrt{2\gamma\cos^2\varepsilon}\right), \text{ BPSK}$$

$$\mu = \left|\frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}\right|, A_{\max} = 1+\mu, A_{\min} = 1-\mu,$$

$$x_{cq}(t) = A_C\sin\phi(t) = A_C[\phi(t) - (1/3!)\phi^3(t) + \ldots]$$

$$x_{ci}(t) = A_C\cos\phi(t) = A_C[1 - (1/2!)\phi^2(t) + \ldots]$$

$$f_s = (1-\alpha)f_x, 0 < \alpha < 1$$

$$\alpha < 1/(2m+1)$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$x_\delta(t) = x(t)s_\delta(t) = x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT_s) = \sum_{k=-\infty}^{\infty} x(kT_s)\delta(t-kT_s)$$

$$N_D = \sigma_D^2 + \sigma_Q^2 = \frac{1}{3q^2} + \frac{4P_e}{3}$$

$$\left(\frac{S}{N}\right)_D = \frac{3q^2}{1+4q^2P_e}S_X,$$

$$\left(\frac{S}{N}\right)_D \approx 3M^{2b}S_x$$

$$b = B_T / W$$

$$B_T \approx vW$$

$$q = M^v$$

$$P_e = Q\left[\sqrt{(S/N)_R}\right], \text{unipolar signaling}$$

$$\mathcal{F}\{\delta(t-t_d)\} = \exp(-j\omega t_d)$$

$$X_C(f) \approx \frac{1}{2}A_C\delta(f-f_C) + \frac{j}{2}A_C\Phi(f-f_C), f > 0$$

$$\Phi(f) = \mathcal{F}[\phi(t)] = \begin{cases} \phi_\Delta X(f), \text{PM} \\ -jf_\Delta X(f)/f, \text{FM} \end{cases}$$

$$p_{A_n}(A_n) = \frac{A_n}{N_R} \exp\left(-A_n^2/2N_R\right) u(A_N)$$

$$\overline{A}_n = \sqrt{\pi N_R/2}, \overline{A_N^2} = 2N_R$$

$$P_{we} \approx P(2,n) \approx n(n-1)\alpha^2/2, \quad \alpha << 1$$

$$v_{bp}(t) = A(t)\cos(\omega_C t + \phi(t))$$

$$V_{lp}(t) = A(t)\exp[\phi(t)]/2$$

$$x_C(t) = A_C \cos[\omega_C t + \phi(t)]$$

$$x_C(t) = v_i(t)\cos(\omega_C t) - v_q(t)\sin(\omega_C t)$$

$$v_i(t) = A(t)\cos(\phi(t)), \ v_q(t) = A(t)\sin(\phi(t))$$

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

$$\eta = r_b/W = \frac{\log_2 M}{1+\beta}$$

$$Q(k) = \frac{1}{\sqrt{2\pi}} \int_k^\infty \exp(-\lambda^2/2)\, d\lambda$$