S-88.4200 Statistical Signal Processing. Final Exam
May 16, 2005

1. Define briefly the following concepts:

(a) State-space model

(b) Influence Function

(c) Innovation

(d) Resolution limit

(e) Sufficient Statistics

(f) Convergence in probability

2. Describe Maximum Likelihood approach for direction of arrival estimation

3. Explain the concepts of Fisher Information and Cramer-Rao Lower Bound. Give examples how they are used.

4. a) Let us assume that we have i.i.d. measurements $y(1), ..., y(N)$, that obey the distribution

$$f(y(i)|\theta) = \theta^2 y(i) e^{-\theta y(i)}, \quad \text{where } y(i) \geq 0, \theta > 0$$

Find the Maximum Likelihood estimator for θ

4. b) Compute the ML-estimate of parameter θ from the independent observations $y(1) = -3.5$, $y(2) = -4.2$ and $y(3) = -5.0$. The probability distribution (with parameter θ) of each measurement is of form

$$f(y(i)) = \begin{cases} y(i) - \theta + 1, & \theta - 1 \leq y(i) \leq \theta \\ -y(i) + \theta + 1, & \theta < y(i) \leq \theta + 1 \\ 0, & otherwise \end{cases}$$

5 (a). Let us assume that we have measuremets $y(1), ..., y(N)$. that obey the distribution

$$f(y(i)|\theta) = \frac{1}{2\pi} e^{-\frac{1}{2}(y(i)-\theta)^2}$$

where θ is unknown parameter with pdf

$$f(\theta) = \frac{1}{\sqrt{\pi}} exp[-\theta^2]$$

Find the mean square estimator for θ.

5 (b). We have made statistically independent measurements $y(1), y(2), \ldots, y(N)$. The joint distribution of random parameter θ and measurements $y(i)$ is given by

$$f(y(i), \theta) = \theta e^{-\theta y(i)}.$$

where $\theta > 0$. Find the MAP estimate of θ.

The Bayes rule is:

$$f(\theta|\mathbf{y}) = \frac{f(\mathbf{y}|\theta) f(\theta)}{f(\mathbf{y})}$$